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Enhancing logistics of intermittent demand items: optimization via simulation based stock control using empirical method

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Abstract: In this paper we examine whether empirical method can replace bootstrapping in intermittent demand stock control based on simulation. Thus, we generate artificial demand data with 30; 50 and 70 % of zero demand periods and simulate reorder point/fixed order quantity inventory control policy using past stock movement simulation and the local search to obtain the optimal trade-off between holding and ordering costs and the required fill rate for order lead time 2; 6; 12 and 18 periods. The outputs from simulation experiments prove that empirical method outperforms bootstrapping in term of the consumption of computational time while maintaining similar ability to overestimate lead time demand. Thus, empirical method can become a suitable substitute of bootstrapping in the local search. Moreover, it can be successfully used to generate an initial reorder point in a more on a one-way neighbourhood search oriented optimization. As empirical method copes both with theoretical and empirical demand distributions and does not require a deciding on number of sampling runs, an optimization of smoothing constants based on a selection of an appropriate accuracy metric, an adoption of a demand classification schemes or a data aggregation it is well predetermined to become an important part of a simulation-optimization software solution focusing on sporadic demand inventory control in large scale real life tasks.

1 Introduction

One of the most important tasks in supply chain management is inventory control. By effectively managing inventory such that total cost of ownership is kept to a minimum, the best inventory control techniques aim to lower supply chain costs. Today's market competitiveness is largely determined by a company's capacity to manage the difficulties of cutting expenses and lead times, raising customer satisfaction standards, and enhancing product quality [1]. The challenge of inventory management is to maintain a sufficient supply of a given good to satisfy an anticipated demand pattern while finding a fair trade-off between the expense of keeping the thing in stock and the potential consequences of running out [2].

Intermittent demand, characterized by sporadic demand arrivals with varying sizes and frequent periods of zero demand, poses a significant challenge in forecasting and stock control [3]. Intermittent demand is a prevalent phenomenon in various industries. Sectors like process industries, aerospace, automotive, IT, and the military often have a significant portion of their inventory value attributed to intermittent demand items, particularly in service and repair parts inventories [4]. Additionally, the after-sale industry heavily relies on items with intermittent demands, underscoring their importance in post-sales service [5].

To guarantee successful and economical operations throughout the supply chain, a variety of optimization approaches, technologies, and risk management measures must be integrated when dealing with this kind of demand pattern. In the literature, parametric time series forecasting based on single exponential smoothing (SES) is considered to represent a mainstream approach [6]. It requires to estimate mean and variance of lead time demand with help of a time-series forecasting method and subsequently use these characteristics as an input to stock management usually aimed at reaching the trade-off between a service level and inventory costs [7]. Time series forecasting techniques are widely used in practice because they are straightforward and simple to use. They mostly rely on historical data and make little effort to determine the factors driving the need for demanded items by including contextual information (e.g., expert assessments, product attributes, maintenance information). As a result, they can be easily automated using data that is readily available in ERP systems and take less work to acquire. However, the major drawbacks of the parametric techniques represent an assumption on a standard demand distribution and also perceiving demand forecasting and inventory control to be two separated stages [8].

That leads to the development of alternative data driven approaches including mainly bootstrapping [9], empirical method [10] and most recently the applications of neural networks [11]. As all these non-parametric approaches do not assume the order lead time demand to follow a particular distribution they are suitable for applications in demand forecasting of items with quite complicated and intriguing patterns. On the other hand these approaches still separate estimating order lead time demand from inventory optimization not taking into account a calculation of economic order quantity. Moreover, time consumption to obtain an estimation of order lead time demand can be excessive when compared to traditional parametric time series methods as non parametric techniques may require repeated sampling or lengthy learning about a demand pattern from historical data making the applications of these methods potentially expensive mainly when speaking about large scale real life tasks [12].

The idea of optimizing both when and how much to order as a conjunctive task represents the core of past stock movement simulation (PSMS) [8]. In PSMS a simulated period is divided into time intervals of equal length and a demanded quantity is assigned for each interval based on either historical real demand data or data derived from a generation technique. For each interval a replenishment, a direct demand satisfaction from available inventory, and an ordering is simulated under the control of a selected inventory control policy. To obtain the optimal combination of control variables for the selected inventory control policy an optimization technique is employed trying to reach the trade-off between the required service level and minimal holding and ordering costs. In the literature this procedure is called optimization via simulation [13]. Optimization via simulation (OvS) provides a versatile and effective way to address complex optimization problems in different domains by integrating simulation models with optimization algorithms to efficiently find optimal solutions [14]. More specifically in this case of multiproduct inventory management, the system design variables are discrete valued, and thus the optimization problems are discrete optimization via simulation (DOvS) problems [15]. For DOvS problems many optimization techniques are available including heuristics such as random search [16] or hill climb [17] and metaheuristics represented for example by evolutionary algorithms [18], tabu search [19] or simulated annealing [20]. In this paper, rather than on the efficient exploring of a solution space we focus on its reduction following the idea of local search (LS) proposed by [8]. These authors pointed out that supplementing PSMS with all combinations search (AC) certainly outperforms parametric forecasting methods in term of reaching lower holding and ordering costs, though it suffers from excessive consumption of computational time for time series with a high total non-zero demand (S). Thus, in LS, [8] underestimate order lead time demand using linear regression (R_{LR}) and overestimate order lead time demand using bootstrapping (R_B) and manage to explore significantly reduced number of $R_B - R_{LR}$ reorder points/fixed order quantity (Q) combinations in (R, Q) inventory control policy bringing substantial savings of the computational time while maintaining pretty decent ability to reach the best possible holding and ordering costs (i.e. the ability to outperform parametric forecasting methods). We continue to develop the principles of LS and examine whether in the overestimating order lead time demand a replacing bootstrapping with empirical method (EM) has a potential to bring additional time savings as bootstrapping is based on tardy repetitive sampling from historical demand data. Thus, we generate artificial intermittent demand data with a different level of sporadicity (i.e. 30; 50 and 70 % of zero demand periods), simulate (R, Q) inventory control policy and compare PSMS+AC, PSMS + LS with LR and B and PSMS + LS with LR and EM in term of the consumption of computational time and trade-off between required fill rate and minimal reached holding and ordering costs for lead times ranging from 2 to 18 periods.

2 Methodology

2.1 *Replacing bootstrapping with empirical method in overestimating lead time demand*

As bootstrapping originally proposed by [21] requires to set a sufficient number of sampling runs (i.e. 100; 1 000; 5 000…) consisting of lead time selections of a demand from historical data to construct an empirical distribution of lead time demand it can be quite time consuming. That is why we suggest to simplify this procedure and employ empirical method by [22] which is also a way easily to understand and implement. Empirical method does not randomly sample demands from a time series, it just gradually sums up these demands according to a lead time and similarly to bootstrapping creates an empirical distribution of lead time demand. If a time series consists for example of 20 periods and order lead time is 2 periods, empirical method creates 10 sums for periods 1+2; 3+4; ….; 19+20, uses these sums to create the distribution of order lead time demands and based on a required service level the reorder point is directly set according to the distribution function. A disadvantage of this method is a potentially low number of lead time demands coming from too short time series or too long lead times and that is why we in this study examine the functionality of empirical method for different order lead times (i.e. 2, 6, 12 and periods).

2.2 *Demand data characteristics*

To compare the performance of bootstrapping and empirical method, we create 3 artificial demand data sets each consisting of 10 000 time series with number of zero demand periods 30; 50 and 70 %. The length of a time series is 36 periods. To generate artificial demand data, we apply the two stage process proposed by [8]. At the first stage we randomly generate non-zero demands per period uniformly distributed between 1 and 30 pieces and then we replace randomly selected non-zero demands with zeros to obtain required level of sporadicity. To classify demand patterns of a time series within the data sets we use average demand interval (ADI) based on equation (1):

$$ADI = \frac{36}{N_{S_t}} \tag{1}$$

where N_{S_t} represents number of non-zero demand periods and squared coefficient of variation (CV^2) based on equation (2):

$$CV^2 = \left(\frac{\sigma_{St}}{\bar{S}_t}\right)^2 \tag{2}$$

where σ_{St} represents non-zero demand standard deviation and $\bar{S}_t$ represents non-zero demand average.

We apply a demand classification scheme described in [23] using *ADI* equal to 0.49 and CV^2 equal to 1.32 decisive values to distinguish among smooth, erratic, intermittent and lumpy demand pattern. Number of time series with the certain demand pattern displays Table 1 together with minimal (S_{min}), maximal (S_{max}) and average total demand (S_{avg}).

Table 1 Features of randomly generated demand data

	Demand pattern						
0 demand periods	Smooth	Intermittent	Erratic	Lumpy	S_{min} [pcs]	S_{max} [pcs]	S_{avg} [pcs]
30 %	0	9 632	0	368	227	547	387
50 %	0	9 414	0	586	152	419	279
70 %	0	8 996	0	1 004	69	278	170

It can be seen in Table 1 that we work predominantly with intermittent demand pattern with increasing number of lumpy time series.

2.3 Past stock movement simulation and arrangement of simulation experiments

To simulate randomly generated data we modify original PSMS+LS Excel VBA code and also original PSMS+AC Excel VBA code described in [8] in a way to measure the consumption of computational time separately for R_{LR}, R_B, and R_{EM} calculations and subsequent exploration of a solution space (Figure 1).

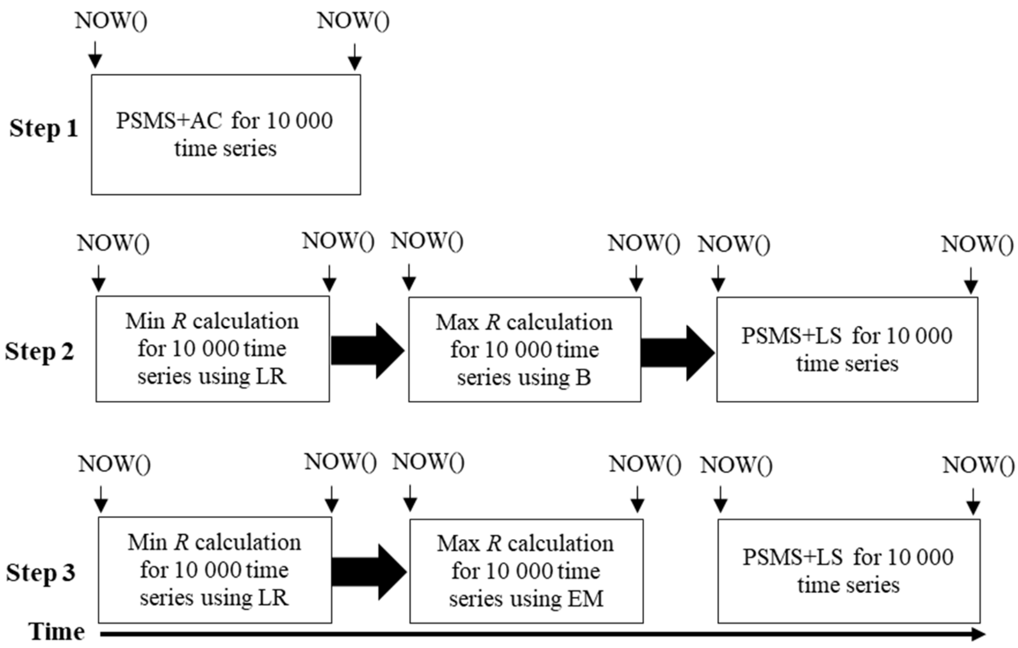

Figure 1 Arrangement of experiments for a data set

We simulate (R, Q) inventory control policy taking into account only such reorder point (R)/fixed order quantity (Q) combinations where $Q > R$. In all simulation experiments we use parameters summarized in Table 2 including holding costs (c_h), ordering costs (c_o), required fill rate (FR) and price (p).

Table 2 Parameters of simulation

c_h [% of average stock in €/period]	4%
c_o [€/1 order]	16
FR [%]	95%
p [€/piece]	70

For a time series and a simulated R, Q combination ensuring at least FR we calculate total holding and ordering costs (C_t) using equation (3):

$$C_t = AI \cdot c_h \cdot p \cdot 36 + N_o \cdot c_o \tag{3}$$

where AI represents average inventory and N_o number of orders.

In agreement with [8] there is no backordering, a partial satisfaction of demand is available and initial inventory is unified in all simulation experiments. Number of sampling runs for bootstrapping is set to be 100. Combining 3 artificial demand data sets (i.e. 10 000 time series each) with PSMS+AC for lead times equal to 2, 6, 12, 18 periods, with original PSMS+LS based on LR and B reorder points estimations for lead times equal to 2, 6, 12, 18 periods and with modified PSMS+LS replacing B with EM again for lead times equal to 2, 6, 12, 18 periods we carry out 3 · 10 000 · 3 · 4 = 360 000 simulation experiments. To execute simulation experiments in MS Excel 2016 environment we use laptop with 2,8 GHz, 16 GB RAM processor.

3 Results and discussion

First, we try to find out whether the empirical method reliably fulfil the role of overestimating order lead time demand and can be therefore an appropriate alternative to bootstrapping. Thus, for every simulated combination of the level of sporadicity (i.e. 30; 50; 70 % of zero demand periods)/order lead time (i.e. 2; 6; 12 and 18 periods) we calculate the differences among reorder points (Δ_R) for a simulated time series in the form of percentiles (Table 3) and also create distributions of reorder points connected with the best reached minimal holding and ordering costs for a simulated time series (Figure 2).

As PSMS+AC returns the best possible holding and ordering costs for a correct function of PSMS+LS based on LR and B or LR and EM we expect $R_{LR} \leq R_{AC} \leq R_B$ or similarly $R_{LR} \leq R_{AC} \leq R_{EM}$. The results in Figure 2 and Table 3 show that overestimated reorder points based on empirical method are distributed closer to the distribution of the best possible reorder points (i.e. R_{AC}s) than reorder points based on bootstrapping. For example in case that level of demand sporadicity is 30 % zero demand periods and lead time is equal to 2 periods the minimal difference R_{EM} - R_{AC} is 1 and 95 % percentile is 30 while the minimal difference R_B - R_{AC} is 4 and 95 % percentile is 32 (see Table 3, rows 4 and 6; red font values). For this combination of the level of demand sporadicity and lead time the local search proposed by [8] performs correctly for the most of simulated time series because minimal R_B - R_{LR} > 0 (see Table 3, row 5) and for at least 95 % of simulated time series R_{LR} – R_{AC} < 0 (see Table 3, row 3). This is in accordance with findings in [8] proving PSMS+LS to work efficiently for smooth/slightly intermittent demand pattern (see Table 1) and we claim also whether order lead time is relatively short. This is because with increasing order lead time the underestimating lead time demand with LR works improperly as for example in case that level of demand sporadicity is 30 % zero demand periods and lead time increases from 2 to 6 periods now for at least 20 % of simulated time series R_{LR} - R_{AC} > 0 (see Table 3, row 8). Moreover, with increasing lead time we also register an occurrence of both R_B - R_{AC} < 0 and R_{EM} - R_{AC} < 0 (see e.g. Table 3, rows 9, 11, 14, 16, 19 and 21; green font values). That brings a potential difficulty to find at least a feasible suboptimal solution with PSMS+LS because order lead time estimations based on LR, B and EM are too low and in many cases they cannot be sufficiently compensate with higher replenishment orders to reach at least required service level. Before we examine this problem closely and summarizes number of simulation experiments where PSMS+LS returns no solution (i.e. results in Table 4) we want to emphasize that the above described inability of LS to work properly continues to deteriorate with increasing level of demand sporadicity when mainly LR is unable to underestimates lead time demand. More specifically, while for the level of demand sporadicity 30 % zero demand periods and order lead time equal to 2 periods R_{LR} – R_{AC} < 0 is reached for at least 95 % of simulated time series, for 50 % zero demand periods it goes down to 90 % (see Table 3, row 23) and for 70 % of zero demands it further decreases to 80 % (see Table 3, row 43). This is mainly because the distribution of R_{AC}s is becoming more volatile and with increasing number of time series with R_{AC} = 1. On the other hand the ability of B and EM to overestimate lead time demand remains pretty decent regardless to the growing level of sporadicity and in case of B it is very stable even for higher lead times. For EM, despite R_{EM} - R_{AC} are in general lower than R_B - R_{AC} with increasing lead time more and more time series tend to R_{EM} - R_{AC} < 0 (see e.g. Table 3, rows 56 or 61) because number of lead time demands coming from time series drops (i.e. from 36/2 = 18; 36/6 = 6; 36/12 = 3 to 36/18 = 2) negatively affecting the ability of EM to build the empirical distribution of lead demand and subsequently to overestimate lead time demand for a required service level successfully.

Table 3 Δ_R percentiles

0 demand periods [%]	Lead time	B/EM	Δ_R - percentiles [%]													Δ_R
			0	**5**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**95**	**100**	
30	2	Both	-26	-19	-18	-16	-15	-14	-12	-11	-10	-8	-5	-2	11	$R_{LR}-R_{AC}$
		B	4	13	15	17	19	20	22	23	25	27	30	32	50	R_B-R_{AC}
			18	27	29	30	32	33	34	35	36	38	39	41	48	R_B-R_{LR}
		EM	1	10	12	14	16	18	19	21	22	24	28	30	52	$R_{EM}-R_{AC}$
			16	23	25	27	29	30	31	33	34	36	38	40	47	$R_{EM}-R_{LR}$
	6	Both	-63	-23	-19	-14	-11	-8	-6	-4	-1	2	7	12	54	$R_{LR}-R_{AC}$
		B	-27	22	27	32	35	38	41	44	47	51	56	62	112	R_B-R_{AC}
			20	35	38	41	43	45	47	49	51	53	57	59	80	R_B-R_{LR}
		EM	-17	10	14	18	22	25	28	31	35	39	46	53	113	$R_{EM}-R_{AC}$
			9	21	23	27	29	32	34	37	39	43	48	52	84	$R_{EM}-R_{LR}$
	12	Both	-72	-29	-21	-13	-7	-3	1	5	10	17	26	34	107	$R_{LR}-R_{AC}$
		B	-30	29	37	46	52	57	62	67	72	79	88	97	161	R_B-R_{AC}
			21	44	47	52	55	58	61	63	66	69	74	78	114	R_B-R_{LR}
		EM	-16	5	11	15	18	23	28	35	42	51	64	76	170	$R_{EM}-R_{AC}$
			5	14	17	21	24	27	30	33	36	41	48	54	112	$R_{EM}-R_{LR}$
	18	Both	-78	-29	-21	-12	-5	1	7	14	22	33	50	64	154	$R_{LR}-R_{AC}$
		B	-17	35	46	58	66	74	81	88	96	107	121	134	219	R_B-R_{AC}
			26	50	54	60	64	67	71	74	78	82	88	93	121	R_B-R_{LR}
		EM	-8	5	11	16	18	20	24	33	43	56	77	95	218	$R_{EM}-R_{AC}$
			-9	7	10	13	16	20	23	26	30	35	42	48	84	$R_{EM}-R_{LR}$
50	2	Both	-29	-20	-18	-16	-14	-12	-11	-9	-7	-5	-2	1	13	$R_{LR}-R_{AC}$
		B	-5	10	12	15	17	19	21	23	25	28	31	34	51	R_B-R_{AC}
			13	23	24	27	29	30	32	33	34	36	39	40	51	R_B-R_{LR}
		EM	-6	7	9	12	14	16	18	19	22	24	28	32	50	$R_{EM}-R_{AC}$
			14	20	21	23	24	26	28	30	32	34	37	39	49	$R_{EM}-R_{LR}$
	6	Both	-66	-24	-19	-13	-10	-7	-5	-2	1	5	12	19	52	$R_{LR}-R_{AC}$
		B	-23	18	23	29	33	36	39	43	47	51	59	65	105	R_B-R_{AC}
			27	32	36	37	40	42	44	46	48	51	54	56	65	R_B-R_{LR}
		EM	-13	7	11	16	19	23	26	29	34	39	48	57	120	$R_{EM}-R_{AC}$
			8	18	20	23	26	29	31	33	36	40	45	50	85	$R_{EM}-R_{LR}$
	12	Both	-71	-27	-20	-11	-6	-1	3	8	14	20	30	39	112	$R_{LR}-R_{AC}$
		B	-21	26	34	43	50	55	60	66	71	79	90	100	174	R_B-R_{AC}
			12	39	43	48	51	54	57	60	62	66	71	75	104	R_B-R_{LR}
		EM	-37	0	7	12	16	22	28	35	42	52	65	77	185	$R_{EM}-R_{AC}$
			0	11	14	17	20	23	26	30	33	38	45	51	96	$R_{EM}-R_{LR}$
	18	Both	-92	-31	-23	-13	-7	-1	5	12	20	32	49	63	147	$R_{LR}-R_{AC}$
		B	-33	29	39	51	60	68	75	82	90	101	116	129	208	R_B-R_{AC}
			16	45	50	55	60	63	67	70	74	78	84	90	120	R_B-R_{LR}
		EM	-9	-1	6	11	12	14	19	27	38	51	73	91	213	$R_{EM}-R_{AC}$
			-10	4	7	10	13	16	20	23	27	32	39	44	95	$R_{EM}-R_{LR}$
70	2	Both	-35	-19	-17	-14	-12	-10	-8	-6	-4	-2	1	3	13	$R_{LR}-R_{AC}$
		B	-16	4	6	9	12	14	16	19	21	23	27	29	50	R_B-R_{AC}
			9	18	19	21	22	23	24	25	26	28	32	35	49	R_B-R_{LR}
		EM	-8	5	7	9	11	13	15	17	19	22	25	28	52	$R_{EM}-R_{AC}$
			9	17	19	20	21	22	23	24	25	27	30	34	49	$R_{EM}-R_{LR}$
	6	Both	-53	-21	-16	-11	-8	-5	-2	1	4	9	16	21	44	$R_{LR}-R_{AC}$
		B	-22	14	19	24	28	32	35	39	43	48	55	61	89	R_B-R_{AC}
			12	25	27	31	33	35	37	39	41	44	48	51	70	R_B-R_{LR}
		EM	-13	4	8	12	16	20	23	27	31	37	46	54	97	$R_{EM}-R_{AC}$
			5	14	16	19	21	23	25	28	30	34	39	43	73	$R_{EM}-R_{LR}$
	12	Both	-57	-21	-15	-8	-3	1	5	10	15	21	31	39	79	$R_{LR}-R_{AC}$
		B	-22	23	30	38	44	49	54	59	65	72	83	91	136	R_B-R_{AC}
			9	31	35	39	42	45	48	51	54	58	63	67	92	R_B-R_{LR}
		EM	-32	-1	5	9	15	21	26	32	39	47	60	72	154	$R_{EM}-R_{AC}$
			-2	8	10	13	16	18	21	24	27	31	37	43	99	$R_{EM}-R_{LR}$
	18	Both	-71	-27	-21	-12	-6	-1	5	11	19	29	44	56	116	$R_{LR}-R_{AC}$
		B	-21	23	32	42	50	57	64	70	78	88	101	113	170	R_B-R_{AC}
			9	36	40	45	49	53	56	60	63	68	74	79	110	R_B-R_{LR}
		EM	-7	-1	3	6	8	9	15	23	31	45	62	79	171	$R_{EM}-R_{AC}$
			-10	2	4	7	9	12	15	18	21	25	31	36	72	$R_{EM}-R_{LR}$

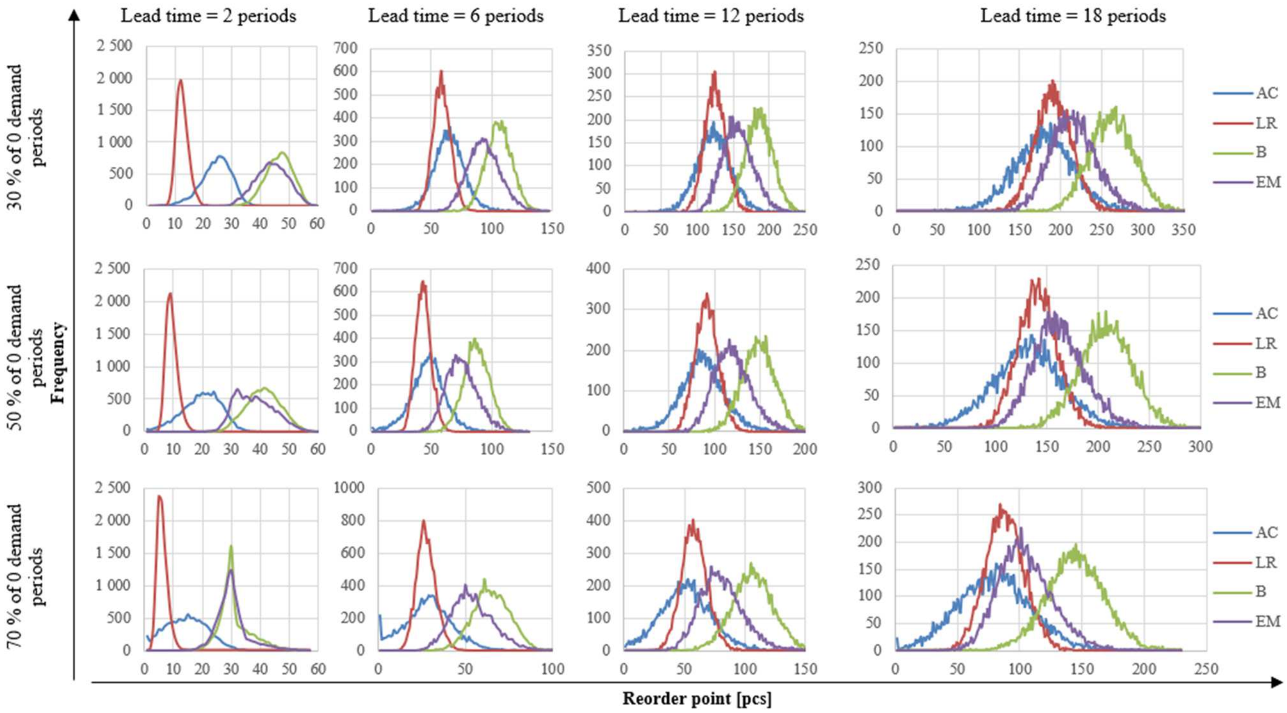

Figure 2 RAC, RLR, RB and REM distribution

Table 4 shows a number of simulation experiments where PSMS+LS returns no solution.

Table 4 Simulation experiments with no solution

0 demand periods [%]	Lead time	B/EM	No solution	No solution for R_B or R_{EM} - R_{AC} < 0	Time series with R_B or R_{EM} - R_{AC} < 0
30	2	B	0	0	0
		EM	0	0	0
	6	B	17	17	19
		EM	42	42	53
	12	B	14	14	18
		EM	161	161	231
	18	B	5	5	19
		EM	282	257	259
50	2	B	0	0	2
		EM	5	5	9
	6	B	35	35	38
		EM	113	113	134
	12	B	27	27	30
		EM	348	348	459
	18	B	4	4	39
		EM	628	530	530
70	2	B	34	34	97
		EM	21	21	49
	6	B	47	47	55
		EM	186	186	241
	12	B	25	25	29
		EM	468	467	607
	18	B	7	7	43
		EM	1 000	789	790

In general there are two reasons why PSMS+LS returns no solution. First, estimations of R_{LR} and R_B or R_{LR} and R_{EM} are both < R_{AC} and at the same time R_B or $R_{EM} \geq R_{LR}$. In this case PSMS+LS returns no solution because there is no examined R/Q combination ensuring to achieve at least the required service level. From the consumption of computational time point of view that means completely wasting time on the generation of R_{LR}, R_B/R_{EM} and also on PSMS+LS searching a fruitless solution space. Second, estimations of R_B or $R_{EM} < R_{LR}$ for example because number of sampling runs for bootstrapping is set too low or because empirical method works with too short time series or too long order lead time. This causes PSMS+LS does not examine a single R/Q combination at all and from the consumption of computational time point of view that means wasting time "just" on the generation of R_{LR}, R_B/R_{EM}. Anyway, the results in Table 4 proves bootstrapping to perform significantly better than empirical method in term of number of simulation experiments where PSMS+LS returns no solution both for increasing level of sporadicity and prolonging lead times. Moreover, empirical method suffers much more from $R_{EM} < R_{LR}$ kind of no solution mainly for too long lead times (see for example Table 4, row 9). PSMS+LS have also a certain ability to overcome low estimated lead time demand through adjusted replenishment orders for both bootstrapping and empirical method. For example for lead time equal to 6 periods from 19 time series with 30 % of zero demands PSMS+LS with B manage to find at least a feasible solution for (19 - 17) = 2 time series (see Table 4, row 4) and PSMS+LS with EM do the same thing for (53 - 42) = 11 time series (see Table 4, row 5) in situation when no $R_B < R_{LR}$ or $R_{EM} < R_{LR}$ takes place.

Besides the distribution of reorder points to compare the ability of bootstrapping and empirical method to overestimate lead time demand in PSMS+LS we also record the consumptions of the computational time separately for the generation of reorder points and to explore a solution space with PSMS+LS. This consumption is shown in Table 5.

Table 5 Consumption of computational time

0 demand periods [%]	Lead time	AC/ LS+B/ LS+EM	LR [min]	B/EM [min]	LS [min]	Total [min]	Simulated combinations	Time consumption of AC or LS [μs/combination]
30	2	AC	-	-	-	43.5	758 009 224	3.44
		LS+B	3.0	16.8	8.3	28.1	126 113 655	3.96
		LS+EM	3.0	4.2	7.8	14.9	117 305 928	3.98
	6	AC	-	-	-	44.8	758 009 224	3.54
		LS+B	3.0	17.0	9.8	29.8	147 896 555	3.97
		LS+EM	3.0	2.1	7.7	12.8	112 706 747	4.08
	12	AC	-	-	-	44.9	758 009 224	3.55
		LS+B	3.0	17.9	9.5	30.4	143 751 137	3.97
		LS+EM	3.0	1.6	5.6	10.3	79 663 608	4.25
	18	AC	-	-	-	45.1	758 009 224	3.57
		LS+B	3.0	18.1	7.9	29.0	115 987 202	4.09
		LS+EM	3.0	1.5	3.6	8.1	46 089 013	4.71
50	2	AC	-	-	-	23.0	394 192 140	3.51
		LS+B	3.0	18.0	5.9	26.8	83 484 157	4.21
		LS+EM	3.0	4.3	5.8	13.1	75 902 081	4.60
	6	AC	-	-	-	24.1	394 192 140	3.67
		LS+B	3.1	18.4	6.8	28.3	97 522 230	4.20
		LS+EM	3.2	2.4	5.3	10.9	72 853 910	4.38
	12	AC	-	-	-	23.9	394 192 140	3.63
		LS+B	3.1	22.4	6.4	32.0	92 541 546	4.18
		LS+EM	3.5	1.9	3.9	9.3	50 095 343	4.65
	18	AC	-	-	-	23.6	394 192 140	3.59
		LS+B	2.9	20.7	5.1	28.7	71 941 595	4.30
		LS+EM	3.0	1.5	2.4	6.9	27 781 994	5.19
70	2	AC	-	-	-	9.5	148 422 089	3.83
		LS+B	3.0	16.8	3.1	22.9	39 756 591	4.70
		LS+EM	3.0	4.0	3.0	10.1	38 448 406	4.72
	6	AC	-	-	-	9.3	148 422 089	3.75
		LS+B	3.0	18.4	3.8	25.1	48 451 721	4.68
		LS+EM	3.0	2.1	3.0	8.1	35 910 839	4.96
	12	AC	-	-	-	9.2	148 422 089	3.71
		LS+B	3.0	18.9	3.5	25.4	44 249 372	4.77
		LS+EM	3.0	1.7	2.2	6.9	23 579 957	5.55
	18	AC	-	-	-	9.3	148 422 089	3.75
		LS+B	2.9	18.7	2.6	24.2	31 498 299	5.03
		LS+EM	3.0	1.5	1.4	5.9	12 397 843	7.01

The difference between the consumption of computational time spent on the generation of R_B and R_{EM} is quite impressive. While in all simulation experiments R_B sampling with 100 runs takes from 16.8 to 22.4 minutes per a data set with 10 000 time series, R_{EM} needs only from 1.5 to 4.3 minutes per a data set with 10 000 time series. Furthermore, in contrary to R_B, the consumption of computational time spent on the generation of R_{EM} decreases with increasing order lead time and from the lead time 6 periods it even takes less time (i.e. from 1.5 to 2.4 minutes a data set with 10 000 time series) than the generation of R_{LR} taking constantly around 3 minutes per a data set with 10 000 time series. It follows that empirical method is not just significantly faster but also more suitable to be applied in tasks where a detailed discretization of time could be advantageous (i.e. switching from months to weeks or days). Together with the significant speeding up of generating the overestimated reorder point, the application of empirical method in PSMS+LS also further reduces an explored solution space through the closer distribution of R_{EM}s due to the distribution of the best possible reorder points coming from PSMS+AC (see Table 5, the column entitled Simulated combinations, LS+B vs LS+EM comparison). This enables to extend the use of PSMS+LS to strongly sporadic demand areas characteristic with lower total demanded quantity as the total consumption of computational time of PSMS+LS longer keeps up to be lower than PSMS+AC (see Table 5, the column entitled Total [min], AC vs LS+EM comparison).

However, the acceleration of the overestimated reorder point generation and the additional reduction of the solution space bringing the time savings must go hand in hand with a corresponding level of holding and ordering costs. That is why we for each simulation experiment calculate the difference between the best reached holding and ordering costs coming from PSMS+LS and the best possible holding and ordering costs coming from PSMS+AC (i.e. $\Delta_{Ct,best}$). These differences are in the form of percentiles displayed in Table 6.

Table 6 $\Delta_{Ct,best}$ percentiles

0 demand periods [%]	Lead time	B/EM	$\Delta_{Ct,best}$ - percentiles [%] 0	10	20	30	40	50	60	70	80	90	95	98	100
30	2	B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	41%
		EM	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	41%
	6	B	0%	0%	0%	0%	0%	0%	0%	0%	3%	8%	14%	22%	122%
		EM	0%	0%	0%	0%	0%	0%	0%	0%	3%	8%	14%	22%	122%
	12	B	0%	0%	0%	0%	0%	1%	4%	8%	13%	21%	29%	40%	103%
		EM	0%	0%	0%	0%	0%	1%	5%	8%	13%	21%	29%	40%	103%
	18	B	0%	0%	0%	0%	1%	3%	6%	9%	15%	23%	32%	43%	95%
		EM	0%	0%	0%	0%	1%	4%	7%	10%	15%	23%	32%	43%	95%
50	2	B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	3%	11%	101%
		EM	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	3%	11%	101%
	6	B	0%	0%	0%	0%	0%	0%	0%	1%	7%	15%	24%	38%	116%
		EM	0%	0%	0%	0%	0%	0%	0%	2%	7%	15%	24%	38%	116%
	12	B	0%	0%	0%	0%	0%	3%	7%	12%	20%	32%	45%	60%	145%
		EM	0%	0%	0%	0%	0%	4%	8%	13%	20%	33%	46%	60%	145%
	18	B	0%	0%	0%	0%	0%	3%	8%	13%	21%	35%	47%	65%	159%
		EM	0%	0%	0%	0%	1%	4%	8%	14%	22%	35%	48%	66%	159%
70	2	B	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	9%	18%	74%
		EM	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	9%	18%	74%
	6	B	0%	0%	0%	0%	0%	0%	1%	7%	16%	30%	46%	68%	517%
		EM	0%	0%	0%	0%	0%	0%	1%	7%	16%	30%	46%	68%	517%
	12	B	0%	0%	0%	0%	2%	7%	13%	22%	34%	55%	76%	104%	368%
		EM	0%	0%	0%	0%	3%	8%	15%	23%	36%	57%	78%	105%	368%
	18	B	0%	0%	0%	0%	0%	5%	11%	21%	35%	58%	84%	112%	279%
		EM	0%	0%	0%	0%	1%	7%	13%	23%	37%	61%	86%	115%	279%

In general, it can be seen in Table 6 that mainly for longer lead times (i.e. 12 and 18 periods) bootstrapping in PSMS+LS performs slightly better than empirical method. For the level of sporadicity 30 % of zero demand periods and the lead time equal to 2 periods PSMS+LS with both B and EM reached the best possible holding and ordering costs for at least 95 % of simulated time series and the maximal difference in the total costs is up to 41 % compared to PSMS+AC. In term of total holding and ordering costs, the ability of PSMS+LS to perform similar to PSMS+AC decreases with increasing number of zero demand periods and also with the prolonging of lead times. This confirms that especially for a demand data with a higher level of sporadicity it is useful to replace the local search with a more on a neighbourhood search oriented optimization based on a generation of a single reorder point. The outputs from the simulation experiments show that empirical method is definitely the number one choice. It outperforms bootstrapping and linear regression in term of the consumption of computational time while maintaining the ability to execute one way exploration of the solution space during the optimization. This is because, similarly to bootstrapping, for the majority of generated data empirical method reliably overestimates lead time demand (i.e. the additional optimization rests in gradually decreasing the reorder point) and in a relatively stray case that the lead time demand is underestimated PSMS+LS returns mostly no solution (i.e. the additional optimization focuses on gradually increasing the reorder point).

4 Conclusions

In this paper we examine whether empirical method can replace bootstrapping in intermittent demand stock control based on simulation. Thus, we generate artificial demand data with 30; 50 and 70 % of zero demand periods and simulate reorder point/fixed order quantity inventory control policy using past stock movement simulation and local search proposed by [8] to obtain the optimal trade-off between holding and ordering costs and the required fill rate for order lead time 2; 6; 12 and 18 periods. The outputs from simulation experiments prove that empirical method outperforms bootstrapping in term of the consumption of computational time while maintaining similar ability to overestimate lead time demand. Thus, empirical method can become a suitable substitute of bootstrapping in the local search. Moreover, it can be successfully used to generate an initial reorder point in a more on a neighbourhood search oriented optimization as it potentially suffers from a less blindness compared to linear regression. Besides additional time savings, optimization via simulation based on a single reorder point generation would also enable to control the consumption of computational time more efficiently and make for example a decision whether for a certain demand data it is advantageous to apply PSMS+AC prior to the optimization. This is because number of simulated R/Q combinations in PSMS+AC is equal to $\frac{Total\ demand \cdot (Total\ demand - 1)}{2}$ and for the generated single reorder point it is then easy to decide on some additional time spent on one way neighbourhood search simply assigning a certain amount of computational time to every change of the initial reorder point. Empirical method does not require any kind of settings such as deciding on number of sampling runs in bootstrapping. It also does not require any kind of optimization of smoothing constants based on a selection of an appropriate accuracy metric which is common for SES based parametric time series forecasting methods or an adoption of demand classification schemes and data aggregation. As an assumption free and data driven nonparametric approach it also copes with both theoretical and empirical distributions of demand. This predetermines empirical method to become an important part of a simulation-optimization software solution focusing on sporadic demand inventory control in large scale real life tasks.

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