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Measuring productivity using Data Envelopment Analysis and Multiple-Objective Programming in flows, logistic and transportation

Maghsoud Amiri
Department of Industrial Management, Faculty of Management and Accounting, Allameh Tabataba'i University, 14348-63111, Tehran, Iran, amiri@atu.ac.ir
Jafar Esmaeeli
Department of Computer Engineering, Technical and Vocational University (TVU), 14357-61137, Tehran, Iran, ej.manager@gmail.com
Mani Sharifi
Department of Information Systems & Analytics, Farmer School of Business, Miami University, Oxford, Ohio, 45056, United States of America, sharifm@miamioh.edu
Shakib Zohrehvandi
New Technologies department, Center for European Studies, Kharazmi University, 15719-14911, Tehran, Iran, shakib.zohrehvandi@srbiau.ac.ir (corresponding author)
Lucia Knapcikova
Faculty of Manufacturing Technologies with a Seat in Prešov, Department of Industrial Engineering and Informatics, The Technical University of Košice, Bayerova 1, 08001, Prešov, Slovak Republic, EU, lucia.knapcikova@tuke.sk

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Abstract: The logistic and transportation plays an integral part in maintaining a well-functioning organization. One of the most extensively used, original, famous, and popular non-parametric methods for evaluating the efficiency of organizations is the Data Envelopment Analysis, DEA technique. Suppose we can formulate the concept of effectiveness in the DEA technique. In that case, we will be able to measure the productivity of organizations since productivity is a blend of efficiency and effectiveness. Several studies have been developed, e.g., the “Malmquist Productivity Index” (MPI) and the “Lunberger Productivity Index” (LPI), which assess the productivity of corporations through the DEA technique, but these models do not display all factors in a system. Also, they need at least two periods to appraise productivity. Furthermore, their two components of efficiency and effectiveness are not considerably evident. Moreover, sensitivity analysis is not possible in these models. Therefore, a model was presented that can measure the relative productivity of decision-making units through the technique of DEA, simultaneously in a period through the two elements of efficiency and effectiveness with the feature of sensitivity analysis and its solution method is more reliable due to the use of multi-objective planning method. In addition, a case study was used to indicate the application of the proposed model, which demonstrated that a branch could be efficient but unproductive.

1 Introduction

Productivity measures provide answers to important questions about the flows, logistic and transportation. For example, how efficiently transportation providers move people and goods, and whether the value of their services has grown more rapidly than the costs of the inputs they use. Evaluating the performance of similar organizations and examining the results of their performance in a certain period is considered an important and strategic process that determines the competitive position of organizations. It has a notable role in continuous improvement and increasing the quality and effectiveness of management decisions of organizations [1].

One of the main criteria for evaluating the performance of organizations is productivity. Research show that productivity is the most favourable criterion of performance in all evaluations. If organizations do not properly assess their productivity, their survival will be accidental [2]. The measurement provides the means to identify effective factors in improving efficiency and effectiveness, which in turn has a special role in determining the productivity of organizations. Productivity can be divided into two elements: efficiency and effectiveness. Efficiency refers to the ability to obtain output from fewer data, and effectiveness refers to matching the results obtained from work with predetermined goals [3]. Productivity is managing the set of activities that are carried out to enhance the efficiency and effectiveness of the companies.

The DEA is one of the most popular and major techniques for evaluating the relative efficiency of “decision-making units” (DMUs) [4]. Clermont and Schaefer state: “From the point of view of many researchers and users of operations research, the advantages of DEA outweigh its disadvantages”. One of the DEA technique's weaknesses is that it only focuses on evaluating the relative efficiency of DMUs. As a result, evaluating productivity through the popular and widely used technique of the DEA can be considered an important step in determining the performance of organizations [5].

Newly, studies have been developed to measure productivity through the DEA technique, e.g., the "Malmquist Productivity Index" (MPI) and "Lunberger Productivity Index" (LPI), which are mentioned below.

2 Literature review

Qu et al. [6], using the "Weighted Entropy Method" in the first step, calculated the comprehensive index of environmental pollution in different sectors and cities. In the second step, through the combined MPI-LPI, they evaluated the efficiency of different sectors and cities in terms of reducing pollution from 2012 to 2017. At the division level, two main factors affecting the total productivity of each division were determined. At the city level, it was determined that five cities have improved in terms of productivity, but the productivity of the three cities has decreased. Bansal et al. [7] proposed a model to evaluate changes in the "Total Factor Productivity" (TFP) of 60 Indian banks using the MPI-LPI from 2013 to 2017. Data had negative values and undesirable characteristics. Results showed that foreign banks fulfil better than their counterparts in terms of productivity. Giacalone et al. [8] used the MPI from 2011 to 2016 to evaluate and analyze the Italian Judicial System's productivity. Empirical studies showed that the inefficiency of the judicial system slows down the economy due to prolonged judgment and lack of legal certainty. Huang et al. [9] analyzed the dynamic trend of energy security performance of 30 Chinese provinces from 2008 to 2017 through the MPI. The results showed that the eastern and southern provinces perform better energy security than the western and northern regions.

Lu et al. [10] used the "Network DEA" (NDEA) technique and the MPI to evaluate the productivity of the machine tool industry in Taiwan during the years 2010 to 2014. The results showed that the productivity in the production and marketing sectors had rapid growth, and the marketing sector's productivity growth was more than that in the production sector. Amiri [11] presented a new approach to assessing the productivity of DMUs through the NDEA. They used "Multi-Objective Programming" (MOP) to solve network problems. Using the two concepts of efficiency and effectiveness in the DEA model is one of the important features of this research. Wang and Feng [12] analyzed the productivity of China's Industrial System and the subdivision of this industry using the NDEA method and the TFP index from 2004 to 2015. They found that the total productivity of China's Industrial System increased during this period. In the first stage, Ding et al. [13] used the NDEA technique to assess the "Circular Industrial Economic System" efficiency. Then they used the MPI technique to measure the dynamic productivity from 2012 to 2017. Their proposed method can decompose the circular economic system into 4 dynamic indicators and provide more details. Tavana et al. [14] utilized fuzzy NDEA and the MPI to dynamically measure the productivity of oil refineries in the existence of adverse outputs during the years 2013 to 2016. The results showed that the productivity score of 70% of refineries is lower than average.

Aduba and Asgari [15] used the MPI method to evaluate the Japanese Manufacturing Industry's TFP, technological, and efficiency changes. The results showed that the Japanese Manufacturing Industry's productivity declined from 2008 to 2014. Li et al. [16] used the MPI method to evaluate TFP from 1978 to 2016 under resource and environmental limitations in China. The results showed that China's TFP relates to fluctuations and rules of macro-economic, direction control, and economic system reform. Using the MPI method, Song et al. [17] investigated Chinese universities' productivity. They found that the productivity of Chinese universities increased between 2009 and 2016. Lu and Xu [18] utilized the three-stage MPI-DEA to measure TFP in provincial water resources in China from 2008 to 2015. They concluded that it is necessary to reform the existing water consumption system by strengthening government macro-control and strengthening efforts to purify pollution and environmental protection. The growth of TFP of water in China has not yet reached maturity. Liang Yang et al. [19] measured productivity changes in Chinese research universities from 2010 to 2013 using the LPI. The experimental results showed that LPI increased significantly during the period under review.

Falavigna et al. [20], in research entitled "DEA-based MPI for understanding courts reform", used a two-stage analysis to assess the productivity of the Italian tax judiciary from 2009 to 2011. The evidence showed that reducing the number of active divisions harmed the courts' productivity. Gandhi and Sharma [21] measured the productivity of private sector hospitals in India using DEA and MPI from 2010 to 2014. The results showed an improvement in the Indian hospital industry during this period. Fujii et al. [22] assessed the changes in productivity and efficiency in EU28 banks from 2005 to 2014 using "Weighted Russell Directional Distance" and the MPI. Then, they analyzed the share of private bank inputs in increasing productivity and efficiency. They concluded that productivity in EU banks is higher than in the old EU. Cadavid et al. [23] evaluated the productivity of public universities in Colombia through DEA and the MPI from 2011 to 2012. Universities were also ranked using a "Pareto Efficient Cross-Efficiency Model". The results showed an improvement in productivity during the mentioned years. Fernandes et al. [24] evaluated the productivity of European domestic banks and estimated the impact of banking risk factors on their performance from 2007 to 2014. The DEA technique used in this research is based on an MPI to calculate banks' productivity scores. The results showed that credit risk and liquidity hurt banks' productivity, and profit and capital risk harm their performance. The results showed that some provinces had faced a decrease in the productivity of production factors. Research on optimization by heuristic algorithms in manufacturing and industries for measuring the efficiency of using optimization methods and techniques has been done so far, which can be mentioned [29].

The summary of these studies is given in Table 1. As can be seen, most of these studies use the MPI or LPI or a combination of these two indexes to evaluate productivity. First, studies of productivity through efficiency and effectiveness are very limited. Secondly, most of this research evaluate the performance of DMUs through DEA and NDEA using indicators such as MPI and LPI, in which the effectiveness is not evident clearly. Thirdly, these studies need at least two periods or two stages, and efficiency and effectiveness must be considered interdependent, which is not considered in these studies. Fourth, these studies do not have the advantage of sensitivity analysis in the model. In the study of Amiri [11], the method of solving their proposed model was done by converting the fractional model to a linear model, which, unlike that, our solution process in this study will be through the MOP technique.

As a result, to complete the defects in the studies, we proposed a model that can assess the productivity of DMUs through the DEA technique in one stage and one period while maintaining efficiency and effectiveness dependency. Also, the method of solving our model is through the MOP technique and can be used for sensitivity analysis and parametric programming. Also, a case study was used to show the utilization of our model.

Table 1 Summary of studies

Reference	Description of the research	Methodology
Qu et al. (2022)	Evaluation of efficiency in pollution of different sectors and cities for reducing pollution in the years 2012 to 2017	MPI-LPI and “Weighted Entropy Method”
Bansal et al. (2022)	A model to evaluate changes in the TFP of 60 Indian banks during the years 2013 to 2017	MPI-LPI
Huang et al. (2021)	Analyze the dynamic trend of energy security performance of 30 Chinese provinces during the years 2008 to 2017	MPI
Lu et al. (2021)	Evaluate the productivity of the machine tool industry in Taiwan during the years 2010 to 2014	NDEA and MPI
Esmaeeli et al. (2021)	A new approach to evaluating the productivity of DMUs	NDEA and MOP
Giacalone et al. (2020)	Evaluate and analyze the productivity of the Italian Judicial System From the years 2011 to 2016.	MPI
Wang and Feng (2020)	Evaluate the productivity of China's Industrial System and subdivision of this industry from 2004 to 2015	TFP index and NDEA
Aduba and Asgari (2020)	Evaluate the TFP changes, technological changes, and efficiency changes in the Japanese manufacturing industry from 2008 to 2014	MPI
Ding et al. (2020)	Measure the productivity of the “Circular Industrial Economic System” and evaluate the dynamic productivity from 2012 to 2017.	NDEA and MPI
Tavana et al. (2019)	Measure the productivity of oil refineries in the existence of adverse outputs during the years 2013 to 2016.	Fuzzy NDEA and MPI
Li et al. (2019)	Analyze the TFP from 1978 to 2016 under resource and environmental limitations in China	MPI
Song et al. (2019)	Investigate the scientific productivity of the Chinese Science System between 2009 and 2016	MPI
Lu and Xu (2019)	Measure the TFP in provincial water resources in China during the years 2008 to 2015	Three-stage MPI-DEA
Liang Yang et al. (2018)	Measure the productivity changes in Chinese research universities from 2010 to 2013	LPI
Falavigna et al. (2018)	A two-stage analysis to assess the productivity of the Italian tax judiciary during the period from 2009 to 2011	MPI
Gandhi and Sharma (2018)	Evaluate the productivity of private sector hospitals in India during the years 2010 to 2014	MPI
Fernandes et al. (2018)	Evaluate the productivity of European domestic banks and estimate the impact of banking risk factors on their performance during the years 2007 to 2014	MPI

Fujii et al. (2018)	Evaluate the changes in productivity and efficiency in EU28 banks during the period from 2005 to 2014 and analyze the share of private bank inputs in growing its productivity	"Weighted Russell Directional Distance" and MPI
Cadavid et al. (2017)	Evaluate the efficiency of public universities in Colombia during the years 2011 to 2012 and rank the Universities	"MPI and Pareto Efficient Cross-Efficiency Model"
Salarieh et al. (2016)	Study the effect of efficiency and technology changes on the changes in the productivity of the agricultural sector in Iran in the period from 2004 to 2013	MPI

3 Efficiency, effectiveness, and productivity

According to Richard Daft, efficiency is the force by which we must reduce costs and increase profits. Peter Drucker called efficiency the ability of a corporation to reach the desired level of outputs with the minimum level of inputs. Nevertheless, efficiency has also been defined as "the ratio between expected consumption resources and actual consumption resources". Efficiency is also defined as the optimal use of resources that leads to customer satisfaction. In addition, efficiency is defined as "the ratio of time dependent on the ideal system to the total time spent" [25]. A company is efficient if it produces the maximum output from a given input level. The common denominator of all the above definitions in the efficiency field is the maximum use of minimum resources, which is the ratio of output to input.

Effectiveness is the second word that is less discussed. A simple and appropriate definition of effectiveness is the organization's ability to achieve preset goals. Such a definition leads to an interesting concept of effectiveness: there is usually no limit to the effectiveness of an organization. Determining effectiveness is more difficult than efficiency because of its definition, which concerns the Relationship between inputs or outputs and outcomes. Figure 1 shows that some outputs and external environmental factors may affect the outcomes. Changes in effectiveness are mainly focused on changes in outcomes. In the same way, effectiveness establishes a Relationship between input and output with the obtained outcome (end goals) [25].

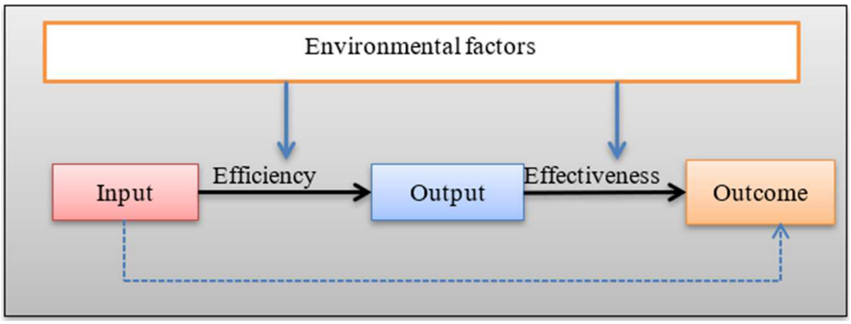

Figure 1 Input, output, and outcome

Effectiveness shows how a good organization works to achieve its goals, but inefficiency is related to waste and, therefore, ineffective operations.

The third word is productivity, which is more comprehensive and complete than the previous two. In the past, the terms productivity and efficiency were used interchangeably, but today these terms have different meanings. Productivity will follow the measurement and evaluation of the output and results of a corporation's activities concerning the goals and the number of consumed resources. Productivity is one of the most important indicators showing employee activities' effectiveness. In 1950, the Organization for "Economic Co-operation and Development" (OECD) defined productivity as "the ratio of output to one of the factors of production". In 1995, three definitions of productivity were shown, which are mentioned below [25]:

Productivity is equal to $\frac{output}{input}$, which is defined as the measurement of efficiency.

Productivity is a blend of efficiency and effectiveness, which is shown as $\frac{output}{input} + \frac{output}{goal}$, which is the concept of productivity [26].

It refers to a broader concept and is anything that improves the organization's performance.

In addition, the "Asian Productivity Organization" (APO) has defined productivity as "$Productivity = efficiency + effectiveness = doing\ things\ right + doing\ the\ right\ things$" [25]. Productivity can also be expressed as "measuring the organization's ability to convert input resources into goods and services". However, the combination of high efficiency and effectiveness in the product production process will lead to high productivity. Therefore, an efficient system may be ineffective, or an effective system may be inefficient. Productivity has been introduced as one of the basic mechanisms for gaining a competitive advantage.

According to the information in this section, we find out that the evaluation of productivity is more complete than the evaluation of efficiency and effectiveness separately. Also, efficiency and effectiveness are two integrated elements of productivity, so using them to measure productivity is better. In addition, according to the information in this section, the productivity formula can be formulated as follows: productivity $= \frac{output}{input} + \frac{output}{goal}$.

4 DEA technique

Farrell [28] proposed non-parametric methods for efficiency estimation for the first time. His model for measuring efficiency included one input and one output. The linear form of their model was called the DEA, and the model they presented was known as the CCR model based on the first letters of their name. The CCR model was changed to a new model known as the BCC model. The DEA is a “Non-Parametric Frontier Evaluation Model” that evaluates the relative efficiency of a set of similar units. This frontier contains line segments that not only identify the most efficient DMUs but also provide an analysis of the inefficient units.

Evaluation of efficiency via the DEA technique is classified by three separate structures: “Overall Technical Efficiency” (OTE), “Pure Technical Efficiency” (PTE), and Scale Efficiency. The Relationship between these three structures is defined as follows (1):

$$\text{scale efficiency} = \frac{\text{OTE}}{\text{PTE}} = \frac{\text{CCR}}{\text{BCC}} = \frac{\text{"Constant Return to Scale" (CRS)}}{\text{"Variable Return to Scale" (VRS)}} \quad (1)$$

The scale efficiency specifies CRS or VRS for greater productivity. OTE, or CCR, is calculated by measuring how well a DMU can use its inputs to create outputs. The CCR model shows that a change in the number of inputs will cause a similar output change. For example, if the input values for a decision unit are doubled, it must produce double the output. PTE or BCC shows that a change in the value of the inputs produces a variable value in the outputs [23]. Usually, the efficiency score of the BCC model is higher than the CCR model.

Furthermore, DEA models need input-oriented and output-oriented solutions to reach the efficient frontier. The input-oriented model provides suggestions on reducing inputs to reach the efficient frontier. For example, an input-oriented efficiency score of 0.9 in an organization indicates that the organization can reduce the input level by 10% with the same output level. The output-oriented model offers suggestions for how to increase output to reach the efficient frontier [21]. In the output-oriented model, it is possible to increase the outputs to reach the efficient frontier without attracting more inputs. The output-oriented model is appropriate where the inputs are approximately constant. Also, the input-oriented model is appropriate when the outputs are closely aligned with the organization's goals or are limited by external factors. If the manager has little control over inputs and most of the managers' operations are to develop outputs, then the output-oriented measurement would be suitable. However, the choice of orientation has little effect on efficiency scores [15]. In general, the benefits of DEA include the following:

There is no need to specify a mathematical function.

It helps discover issues that are hidden from other techniques.

Inefficient resources can be analyzed for each DMU.

A DMU is directly compared to its peers by multiple inputs and outputs.

It uses quantitative and qualitative criteria to evaluate the efficiency of organizations.

The impact of subjective factors is reduced in this method [19].

Other measurement methods measure the company's performance only from a financial landscape. Still, the DEA method measures the company's performance both from a financial landscape and from a non-financial landscape.

The DEA technique allows each DMU to set its variable weight more favourably than other DMUs and can identify reference units for each DMU.

DEA is more flexible and applicable than other methods.

Based on the information given in this section, the reason for choosing the DEA method to evaluate the productivity of DMUs is determined. Also, the type of DEA model is obtained.

5 Productivity indexes

Measuring the index involves using five ratios to measure productivity: “Single Factor Productivity, Multi-Factor Productivity, TFP, Management Control Ratio, and Productivity Costing”. The most common ratio is the TFP, where productivity is measured as a ratio of different inputs. These indicators have recently been combined with DEA and show changes in efficiency over time, and for this reason, they are interpreted as productivity indicators. One type of TFP index combined with DEA is the MPI. The MPI is an index that shows the growth of the TFP of an organization, and it can provide progress or regression in efficiency and show the changes in efficiency between two time periods. If the MPI value is higher than 1, it indicates that the efficiency is improved [20]. The required values of the MPI have been calculated using the DEA technique. By applying it, the changes in total productivity are calculated by separating the changes in efficiency and technological changes. In fact, in the MPI, there is no cost minimization or income maximization assumption, and we only need to observe some inputs and outputs [3].

The MPI is related to “Russell's Measure of Inefficiency”, which is multiplicative, while LPI is based on a “Slack-Based Measurement of Efficiency”, which is cumulative. The DEA technique measures the efficiency of corporations for a specific year. The MPI or LPI approach is used to get changes in efficiency in more than one year [21].

Many productivity studies have recently been developed based on DEA-LPI and DEA-MPI techniques. When the DMUs follow homogeneous production technologies within groups but are compatible with heterogeneous production technologies at the whole society's level, the MPI-LPI scores and their components may be misleading [27].

In this part, we find that these indicators need at least two periods to analyze the efficiency of organizations, the effective formula is not evident in them, and sensitivity analysis is not possible in these models.

6 Multiple-Objective Programming technique

The Multiple-Objective Programming, MOP technique is done by a set of objective functions that must be optimized simultaneously, and a set of constraints is defined to be satisfied. In other words, MOP shows how to move toward several objectives simultaneously.

Considering that in this research, we will use two objective functions of efficiency and the objective function of effectiveness, the linear programming of this model will be multi-objective.

7 Methodology

According to section (4), the input oriented CCR modeling of the DEA technique is as follows (2):

$$\begin{aligned}
&\mathrm{MaxE}_0 = \frac{\sum_{r=1}^{s} u_r Y_{r0}}{\sum_{i=1}^{m} v_i X_{i0}} \\
&\mathrm{S.t:} \\
&\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \le 1; \quad j = 1,2,\dots,n \\
&u_r, v_i \ge \varepsilon; \quad r = 1,2,\dots,s, \quad i = 1,2,\dots,m
\end{aligned} \tag{2}$$

However, as discussed in the previous sections, Equation (2) only evaluates the efficiency of DMUs. To evaluate the productivity of DMUs, which is more comprehensive than the efficiency evaluation, it is necessary to formulate the effectiveness in Equation (2). Referring to section (4), the effectiveness can be obtained as $\frac{output}{goal}$.

To combine this formula with Equation (2), we need to define the effectiveness of the DMU as follows (3):

$$\begin{aligned}
&\mathit{Effectiveness\ of\ DMU_0} \\
&= \frac{\mathit{weighted\ outputs\ of\ DMU}}{\mathit{weighted\ standard\ outputs\ (goals) of\ DMU}} \\
&= \frac{\sum_{r=1}^{rk} u_r Y_{r0}}{\sum_{r=1}^{rk} \eta_r g_{r0}}
\end{aligned} \tag{3}$$

Where $\eta_r g_{r0}$ is weighted standard outputs (goals) of DMU0.

Now, according to section (4), where $productivity = efficiency + effectiveness$, we will combine Equations (2) and (3). Therefore, if we combine Equation (3) with the objective function of Equation (2), we can evaluate productivity through Equation (4).

$$\begin{aligned}
&MaxE_0 = \frac{\sum_{r=1}^{s} u_r Y_{r0}}{\sum_{i=1}^{m} v_i X_{i0}} \\
&MaxE1_0 = \frac{\sum_{r=1}^{rk} u_r Y_{r0}}{\sum_{r=1}^{rk} \eta_r g_{r0}} \\
&S.t: \\
&\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \le 1; \quad j = 1,\dots,n \\
&\frac{\sum_{r=1}^{rk} u_r Y_{rj}}{\sum_{r=1}^{rk} \eta_r g_{rj}} \le 1; \quad j = 1,\dots,n \\
&u_r, v_i, g_r \ge \varepsilon; \quad r = 1,\dots,s, \quad i = 1,\dots,m
\end{aligned} \tag{4}$$

Equation (4) is in the form of a linear fraction, which can be converted into the linear model through Equation (5) as follows:

$$\begin{aligned}
&MaxE_0 = \sum_{r=1}^{s} u_r Y_{r0} \\
&MaxE1_0 = \sum_{r=1}^{rk} u_r Y_{r0} \\
&S.t: \\
&\sum_{i=1}^{m} v_i X_{i0} = 1 \\
&\sum_{r=1}^{rk} \eta_r g_{r0} = 1 \\
&\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0; \quad j = 1,\dots,n \\
&\sum_{r=1}^{rk} u_r Y_{rj} - \sum_{r=1}^{rk} \eta_r g_{rj} \le 0; \quad j = 1,\dots,n \\
&u_r, v_i, g_r \ge \varepsilon; \quad r = 1,\dots,s, \quad i = 1,\dots,m
\end{aligned} \tag{5}$$

The following process has been developed to solve Equation (5):

Step 1: Every objective is optimized independently of other objectives. For DMU0, we maximize E0 individually to determine its ideal objective values E_0'',

Step 2: Every objective is computed oppositely regardless of the other objective. We minimize E0 to determine its anti-ideal solution $E_0^{-}{''}$,

Step 3: Define the membership function of every objective by its ideal and anti-ideal solutions as $uE_0(E_0) = \frac{(E_0 - E_0^-)}{(E_0^* - E_0^-)}$.

Step 4: Maximize the minimal membership function for all objectives as Equation (6).

$$\begin{aligned} &Max\ = \alpha \\ &S.t: \\ &\alpha \leq\ uE_0(E_0) \\ &in\ addition\ to\ all\ original\ constrains\ in\ eq\iota \end{aligned} \qquad (6)$$

So, α is the minimum of all member functions that are maximized. The overall score $E_0 = \sum_{k=1}^{K}(W_k E_0^k)$ is evaluated for the DMU0.

Equation (5) evaluates the productivity of DMUs through the technique of DEA and MOP, both of which have maximum validity and popularity. We can easily continuously evaluate the productivity of DMUs in one stage, in one period, and with the property of sensitivity analysis through Equation (5). The DMU will be productive if the equation (5) answer equals 2.

8 Case study

In this section, to apply Equation (5), we selected the branches of a bank to evaluate productivity. Most of the bank's activities are related to the flows, logistics and transportation industries. Collecting and attracting all kinds of deposits and allocating them to meet the financial needs of all economic activities is one of the most important banking operations. Analyzing the productivity of the banking industry and determining methods for this purpose is of interest to managers, politicians, economists, and academic researchers [24]. The DEA technique is accepted for performance evaluation in the banking industry. There are rich and extensive studies to assess performance in the banking industry, most of which used the DEA technique.

According to the research done by Amiri [11], the input and output processes in the banking industry were drawn in Figure 2, and its data was set in Table 2.

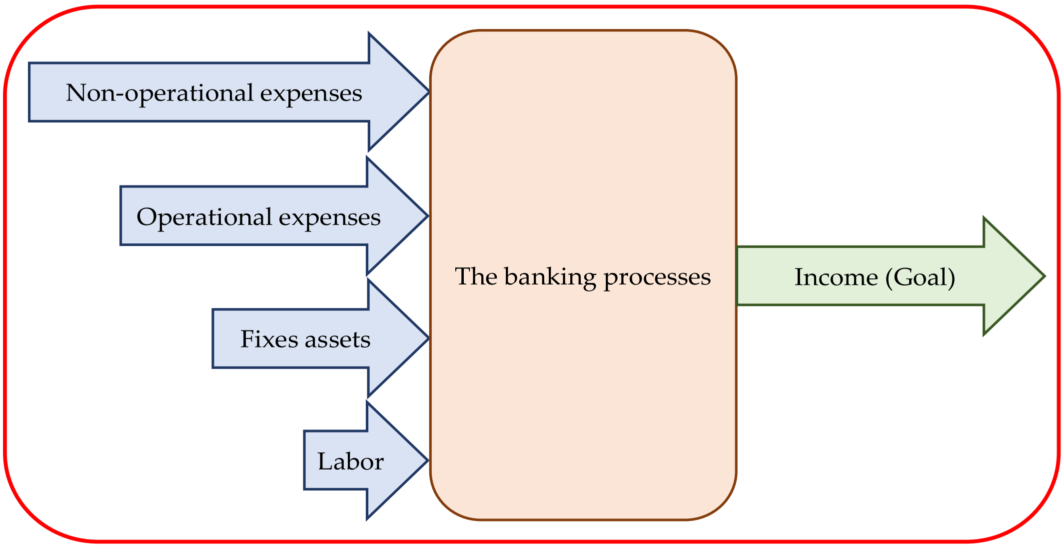

Figure 2 Input and output process in the banking industry

Table 2 Inputs, outputs, and goals

DMUs	Inputs				Output	
	Input1	Input2	Input3	Input4	Output	Goal
1	0.0634	0.0616	0.0378	0.0794	0.0607	0.0729
2	0.0563	0.0907	0.0383	0.0799	0.0463	0.0617
3	0.0563	0.0896	0.0243	0.0383	0.0253	0.0347
4	0.0563	0.0407	0.0277	0.0837	0.0462	0.0581
5	0.0634	0.0435	0.0389	0.0657	0.0435	0.0581
6	0.0563	0.0819	0.0299	0.0681	0.0400	0.0534
7	0.0563	0.0717	0.0338	0.0575	0.0436	0.0576
8	0.0634	0.0448	0.0307	0.0556	0.0408	0.0518
9	0.0775	0.0582	0.0465	0.0629	0.0466	0.0605
10	0.0493	0.0489	0.0295	0.0692	0.0433	0.0538
11	0.0563	0.1120	0.0280	0.0461	0.0298	0.0398
12	0.0563	0.0183	0.0273	0.0577	0.0540	0.0578
13	0.0352	0.0367	0.0151	0.0174	0.0123	0.0173
14	0.0563	0.0326	0.4855	0.0204	0.3336	0.1547
15	0.0563	0.0403	0.0352	0.0711	0.0441	0.0527
16	0.0845	0.0815	0.0457	0.0918	0.0643	0.0818
17	0.0563	0.0469	0.0259	0.0350	0.0256	0.0333

8.1 The results of measuring the efficiency of bank branches through Equation (2)

Using Equation (2), the efficiency scores of the branches are displayed in Table 3 by LINGO software. The results show that branches 12 and 14 are efficient.

Table 3 Efficiency scores of the branches by Equation (2)

DMUs	Efficiency	Rank
1	0.9321776	3
2	0.7626125	7
3	0.6336423	15
4	0.8517058	5
5	0.6612190	13
6	0.7195985	11
7	0.7521600	8
8	0.7426476	10
9	0.6663757	12
10	0.8540485	4
11	0.6309013	16
12	1.000000	1
13	0.5906874	17
14	1.000000	1
15	0.7497362	9
16	0.7660993	6
17	0.6576079	14

8.2 The results of measuring the productivity of bank branches using Equation (5)

Bank branch productivity scores are presented in Table 4 using LINGO software's Equation (5). The results show that branch 14 is productive.

Figure 3 shows the combination of Tables 2 and 3. Considering Figure 3, we find that unit 12 is efficient but unproductive. Also, the rating of some units in Equation (2) and Equation (3) is not the same.

Table 4 Productivity scores of the branches through Equation (4)

DMUs	Productivity	Rank
1	1.3182823	3
2	1.110597	8
3	0.9717502	16
4	1.2204542	5
5	1.0084171	14
6	1.0669611	11
7	1.1031771	10
8	1.1079013	9
9	1.023562	12
10	1.2272728	4
11	0.9781155	15
12	1.4332416	2
13	0.9203907	17
14	2.000000	1
15	1.1377903	6
16	1.1306198	7
17	1.0141083	13

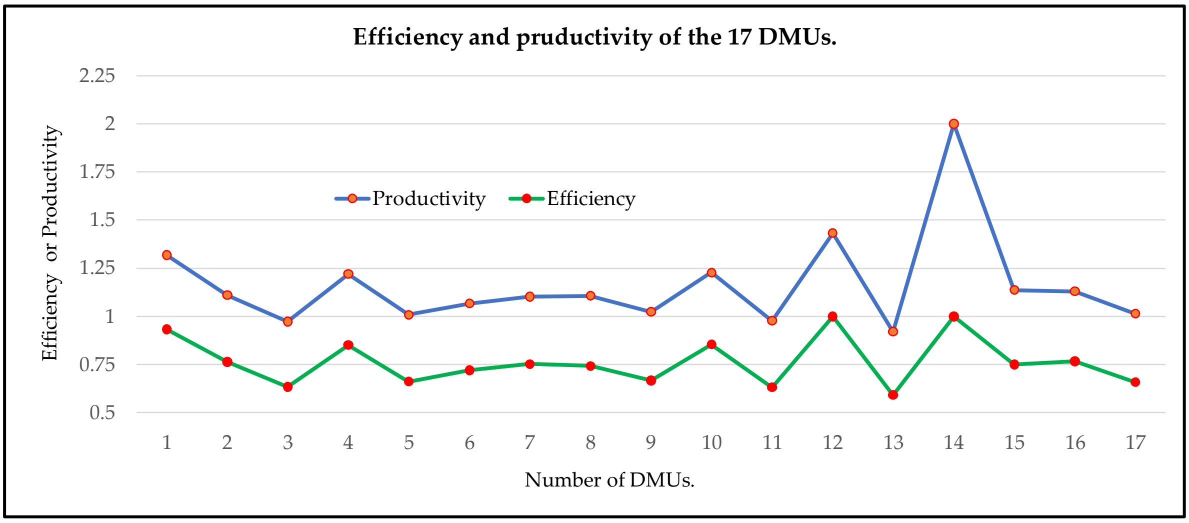

Figure 3 Efficiency and productivity of the 17 DMUs

9 Conclusion and recommendations

Evaluating similar organizations' performance and examining their performance in a certain period is considered an important and strategic process. One of the main criteria for measuring the performance of organizations is productivity. Productivity is a blend of effectiveness and efficiency. The DEA technique is one of the most popular methods and one of the main methods for evaluating the relative efficiency of DMUs. Recently, there have been studies to measure productivity through the DEA technique, which includes the MPI and the LPI. First, the effectiveness is not formulated in these studies. Secondly, they need at least two time periods or two stages. Thirdly, they do not benefit from sensitivity analysis in the model. To complete the shortcomings in the studies, a model was proposed that can assess the productivity of DMUs through the DEA technique and MOP in one stage and one period while maintaining the efficiency and effectiveness dependency. The proposed model is linear in such a way that the advantage of its linearity can be used for sensitivity analysis and parametric programming. For this purpose, effectiveness was formulated in the CCR model of the DEA technique, and MOP was used to maximize the two functions of efficiency and effectiveness. Then, to demonstrate the application of the model, a case study was conducted in the branches of a bank. The case study results showed that a productive branch must be efficient, but an efficient branch is not necessarily productive. Organizations with the same inputs and outputs can benefit from this study to evaluate their productivity, identify less productive resources, and manage their productivity. A DMU can include its predetermined goals in its productivity calculations through the proposed model. Also, a DMU can use sensitivity analysis in the model to determine what change in its productivity will be achieved by changing its predetermined goals. Also, the results showed that the use of this research can play a significant role in flows, logistic and transportation.

Some studies that can be suggested for the future include the following:

- The model's parameters are all precise and deterministic, which can be considered imprecise and qualitative and solved through the fuzzy technique.
- Considering that the proposed DEA model is of CCR type, another research of BCC type can be made.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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