Acta Logistica - International Scientific Journal about Logistics
Volume: 3 2016 Issue: 4 Pages: 7-15 ISSN 1339-5629

DOI: 10.22306/al.v3i4.72
Received: 03 Dec. 2016
Accepted: 11 Dec. 2016

THE ANALYSIS OF THE COMMODITY PRICE FORECASTING SUCCESS CONSIDERING DIFFERENT NUMERICAL MODELS SENSITIVITY TO PROGNOSIS ERROR

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Keywords: exponential approximation, numerical modelling, price forecasting, commodity exchange
Abstract: In the paper the numerical model based on the exponential approximation of commodity stock exchanges was derived. The price prognoses of aluminium on the London Metal Exchange were determined as numerical solution of the Cauchy initial problem for the 1st order ordinary differential equation. To make the numerical model more accurate the idea of the modification of the initial condition value by the stock exchange was realized. The derived numerical model was observed to determine the accuracy of forecast prices with regard to two size setting of the limiting value error causing the modification of the initial condition value by chosen stock exchange. The advantage of chosen sizes of the limiting value error 7 % and 8 % with regard to different lengths of the initial condition drift within movements of aluminium prices was studied. By having analyzed obtained results, it was found out that the limiting value error 7 % was more advantageous for commodity price forecasting.

1 Introduction

Observing trends and forecasting movements of metal prices is still a current problem. There are a lot of approaches to forecasting price movements. Some of them are based on mathematical models [1], [2], [3], [4], [5], [6], [8]. Forecasting prices on commodity exchanges often uses the statistical methods that need to process a large number of historical market data [1], [8]. The quantity of needed market data can sometimes be a disadvantage.

In our prognostic models numerical methods were used. Derived numerical models for forecasting prices were based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations [3], [4], [5], [6].

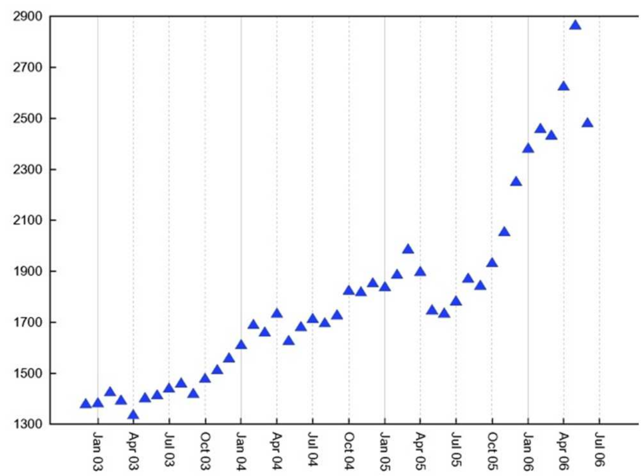

Figure 1 Course of the aluminium prices on LME in the years 2003 – 2006

The aluminium prices presented on the London Metal Exchange (LME) were worked on. We dealt with the monthly averages of the daily closing aluminium prices "Cash Seller&Settlement price" in the period from December 2002 to June 2006. The market data were obtained from the official web page of the London Metal Exchange [9]. The course of the aluminium prices on LME (in US dollars per tonne) within the observing period is presented in Figure 1.

2 Mathematical model

We considered the Cauchy initial problem in the form

$$y' = a_1 y, \ \ y(x_0) = y_0 . \qquad (1)$$

The particular solution of the problem (1) is in the form $y = k\, e^{a_1 x}$, where $k = y_0\, e^{-a_1 x_0}$. The considered exponential trend was chosen according to the test criterion of the time series' trend suitability. The values $\ln(Y_{i+1}) - \ln(Y_i)$, for $i = 0, 1, ..., 42$ have approximately constant course, where Y_i is the aluminium price (stock exchange) on LME in the month x_i. The price prognoses were created by the following steps:

The 1st step: Approximation of the values – the values of the approximation term were approximated by the least squares method. The exponential function in the form $\tilde{y} = a_0 e^{a_1 x}$ was used. When observing the influence of the approximation term length on the prognoses accuracy, we found out that the prognoses obtained by longer approximation terms are more accurate [3]. Let us consider two different variants.

Variant B: The values from the period January 2003 –

June 2003 were approximated. The next approximation terms were created by sequential extension of this period by 3 months. Thus the duration of the approximation terms was extended (the n^{th} approximation term has $6+3(n-1)$ stock exchanges) (Figure 2).

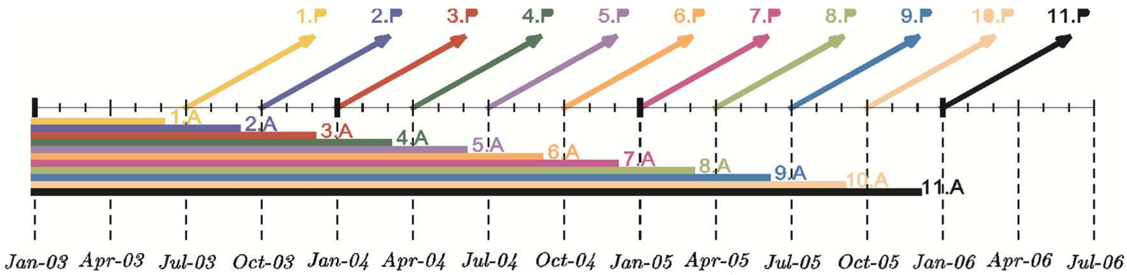

Figure 2 Variant B (A – approximation term, P – forecasting term)

Variant E: We approximated values within 12 months and each term was shifted by 1 month (Figure 3). (The first approximation term was January 2003 – December 2003.)

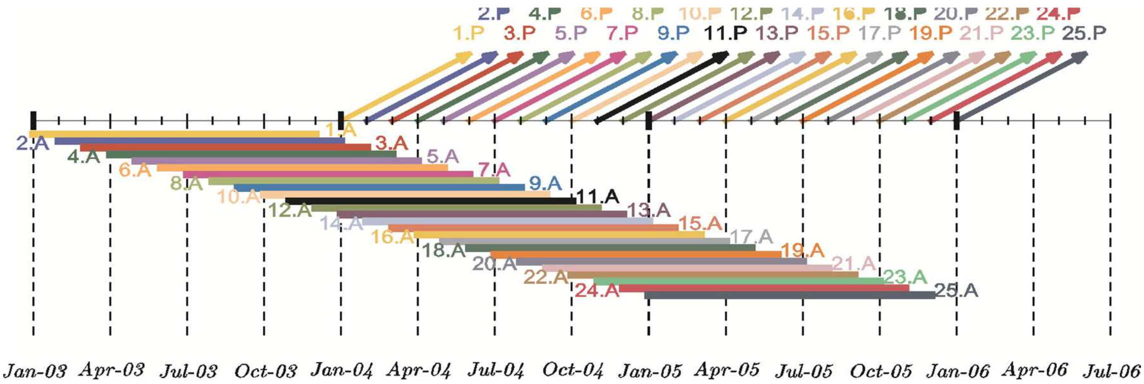

Figure 3 Variant E (A – approximation term, P – forecasting term)

36 forecasting terms of the original model in both variants B and E were observed. From among all forecasting terms, 11 of them belonged to variant B and 25 ones were part of variant E.

The 2nd step: Formulating the Cauchy initial problem – according to the acquired approximation function $\tilde{y}$, the Cauchy initial problem (1) was written in the form

$$y' = a_1 y, \quad y(x_i) = Y_i, \tag{2}$$

where $x_i = i$ and Y_i is the aluminium price on LME in the month x_i, which is the last month of the approximation term.

The 3rd step: Computing the prognoses – the formulated Cauchy initial problem (2) was solved by the numerical method based on the exponential approximation of the solution. A detailed solution method is seen in [7]. The method uses the following numerical formulae

$$x_{i+1} = x_i + h,$$

$$y_{i+1} = y_i + bh + Qe^{vx_i}\left(e^{vh} - 1\right),$$

for $i = 1, 2, 3, \ldots$, where $h = x_{i+1} - x_i$ is the constant size step. The unknown coefficients are calculated by means of these formulae $v = \dfrac{f''(x_i, y_i)}{f'(x_i, y_i)}$,

$$Q = \frac{f'(x_i, y_i) - f''(x_i, y_i)}{(1-v)\, v^2 e^{vx_i}}, \quad b = f(x_i, y_i) - \frac{f'(x_i, y_i)}{v}.$$

If we consider the Cauchy initial problem (2), the function $f(x_i, y_i)$ has the form $f(x_i, y_i) = a_1 y_i$ and then $f'(x_i, y_i) = a_1 y'(x_i) = a_1{}^2 y_i$,

$f''(x_i, y_i) = \; = a_1{}^2 y'(x_i) = a_1{}^3 y_i$.

We calculated the prognoses within six months that follow the end of the approximation term in this way:

- **The first month prognosis** was determined by solving the Cauchy initial problem in the form (2). The interval $\langle x_i, x_{i+1} \rangle$ of the length $h = 1$ month was divided into n parts, where n is the number of trading days on LME in the month x_{i+1}. We got the sequence of the division points $x_{i0} = x_i$, $x_{ij} = x_i + \dfrac{h}{n} j$, for $j = 1, 2, \ldots, n$, where $x_{in} = x_{i+1}$. For each point of the subdivision of the interval, the Cauchy initial problem in the form (2) was solved by the chosen numerical method. In this way we obtained the prognoses of the aluminium prices on single trading days y_{ij}. By calculating the arithmetic mean of the daily prognoses we obtained the monthly prognosis of the aluminium price in the month x_{i+1}.

So, $y_{i+1} = \frac{\sum_{j=1}^{n} y_{ij}}{n}$.

- **The prognoses for the following five months** were calculated after modification of the initial condition value. The initial condition value in the month x_{i+s}, $s = 1, 2, 3, 4, 5$ was replaced either by the calculated monthly prognosis y_{i+s} (the original model) or in case of higher absolute percentage error of given monthly prognosis y_{i+s} by some aluminium stock exchange (the modified model). The Cauchy initial problem $y' = a_1 y$, $y(x_{i+s}) = y_{i+s}$, or $y' = a_1 y$, $y(x_{i+s}) = Y_p$ (where Y_p is chosen aluminium stock exchange) was used for calculating daily prognoses and their arithmetic mean served to define the monthly price prognosis y_{i+s+1} for the month x_{i+s+1}.

By comparing the calculated prognosis y_s in the month x_s with the real stock exchange Y_s, the absolute percentage error $|p_s| = \frac{|y_s - Y_s|}{Y_s}.100\,\%$ was determined. The price prognosis y_s in the month x_s is acceptable in practice, if $|p_s| < 10\,\%$. Otherwise, it is called the critical forecasting value of. To compare the accuracy of the forecasting of all forecasting terms, the mean absolute percentage error (MAPE) $\bar{p} = \frac{\sum_{s=1}^{t} |p_s|}{t}$ was determined, where, in our case, $t = 6$.

The modification of the initial condition value by the real aluminium stock exchange price was called the initial condition drift. Let us name the selected minimal absolute percentage error of the prognosis, causing the initial condition drift, the limiting value error. The month in which the absolute percentage error of the prognosis had at least the limiting value error was considered as the limiting month.

The limiting value errors 7 % and 8 % were chosen. Three types of the initial condition drift with regard to their length were considered, namely one-month drift, drift before the limiting month and drift to the limiting month. One-month drift was the shortest chosen initial condition drift, where the initial condition value was replaced by the stock exchange Y_{i+p}, $p = 1, 2, 3, 4, 5$ in the month x_{i+p}, where x_i was the last month of the approximation term and p was the initial condition drift order in the forecasting term. Using drift before the limiting month, the initial condition value was replaced by the stock exchange Y_{L-1} in the month x_{L-1}, where x_L was the limiting month and by means of drift to the limiting month the stock exchange Y_L in the month x_L changed the initial condition value.

3 Results

3.1 The success rate of the chosen sizes of the limiting value error at the commodity price forecasting

Within the studied group of 36 forecasting terms, the forecasting within 14 of them was so accurate that the initial condition drift did not occur. The initial condition values were replaced just by calculated monthly prognoses. Since in the remaining 22 forecasting terms the forecasting was less accurate, some of the prognoses gained the absolute percentage error higher than chosen limiting value error, and the initial condition drift occurred. Therefore the forecasting results differ from the original model.

We observed the accuracy of the forecast prices with regard to two different size setting of the limiting value error. Within each forecasting term three different lengths of the initial condition drift were taken into account. For each length of the initial condition drift we defined the limiting value error, 7 % or 8 %, to obtain the most accurate forecasting results (the lowest MAPE of the forecasting term). Thus, we considered 66 groups of the forecasting results. The following tables show the number of the forecasting terms in which the forecasting by the determined types of the initial condition drift was the most accurate.

Table 1 The success rate of chosen types of the initial condition drift – variant B

Initial condition drift's length	Identical results for both chosen limiting value errors	More accurate forecasting by means of limiting value error	
		7 %	8 %
One-month drift	3	4	0
Drift before the limiting month	4	3	0
Drift to the limiting month	2	4	1
Total	9	11	1

Table 2 The success rate of chosen types of the initial condition drift – variant E

Initial condition drift's length	Identical results for both chosen limiting value errors	More accurate forecasting by means of limiting value error	
		7 %	8 %
One-month drift	9	3	3
Drift before the limiting month	10	4	1
Drift to the limiting month	7	7	1
Total	26	14	5

The tables clearly show that the forecasting results differed at the determined limiting value errors, and that more accurate prognoses were gained by means of the lower value 7 %. This limiting value error was more advantageous for the forecasting. In the variant B, the success rate of the limiting value error 7 % (11 times) was comparable with the appearance of indentical results for both chosen limiting value errors (9 times). In the variant E within the forecasting terms in which the initial condition drift occurred (15 terms), identical results for both chosen limiting value errors were the most often found (26 times).

The observed variants B and E had different lengths of the approximation terms. In the variant B, the approximation terms were longer, so the forecasting did not react so strongly to fluctuations in the price evolution. Moderately increasing prognoses often acquired the absolute percentage error from the interval ⟨7%, 8%⟩, so the forecasting results with regard to the chosen limiting value errors differed. The lower limiting value error initialized the initial condition drift earlier, and the forecasting was more accurate. On the contrary, the forecasting in the variant E was based on shorter approximation terms which steeply responded to changes in the price evolution. Steeply increasing or decreasing prognoses often obtained, especially within the changes in price course, the absolute percentage error ≥ 8%. Thus, the forecasting results were often identical for both chosen limiting value errors.

The results were more obvious if for each forecasting term, in which the initial condition drift occurred, the most accurate type of the drift's length was considered. For this length of the initial condition drift we defined the limiting value errors 7 % or 8 %, for which the lowest MAPE of the observed forecasting term was obtained (Table 3).

Table 3 The success rate of the chosen types of the initial condition drift for the most accurate length of the drift in the observed forecasting term

Variant	Identical results for both chosen limiting value errors	More accurate forecasting by means of limiting value error	
		7 %	8 %
B	3	4	0
E	8	7	0
Total	11	11	0

If in the forecasting term the most successful initial condition drift's length was considered, the limiting value error 7 % was more advantageous for the forecasting. When it used, the prognosis accuracy was always higher than at forecasting by means of the limiting value error 8 %. The identical forecasting results for both chosen limiting value errors were achieved with the same success as when using the limiting value error 7 %.

The success rate of the determined sizes of the limiting value error was analyzed within different moves of the aluminium price course and was demonstrated at the specific forecasting terms while considering commodity price evolution.

3.2 The forecasting success of lower limiting value error 7 % at the commodity price forecasting

The limiting value error 7 % was more advantageous than higher limiting value error 8 %. It acquired the most accurate results within most price movements. We recommend to use it, especially within the stable increasing price course, and also when the price evolution changes significantly.

- **stable price increase**

Within the stable price course the forecasting by means of the limiting value error 7 % was always more accurate than the forecasting at the limiting value error 8 %. Higher forecasting success at the limiting value error 7 % was achieved by a larger number of the initial condition drift. Thereby the prognoses could better approach increasing stock exchanges, and the forecasting became more accurate. Another advantage of lower limiting value error 7 % is the fact that the next drift occurred earlier than at the limiting value error 8 %.

If the increase was moderate, the prognoses errors were lower. Thereby the initial condition drift occurred only at the limiting value error 7 % at all drift's lenghts. Within a steep increase, the forecasting using the longest initial condition drift was the most accurate [4]. Thereby the absolute percentage prognosis errors were mostly from the interval ⟨7%, 8%⟩, and the initial condition drift occurred only at the limiting value error 7 %. At shorter initial condition drifts, the initial condition value was replaced by stock exchange price that was more distant from real increasing prices. Thereby the prognosis error was higher. Thus, the size of the absolute percentage error often caused initialization of the initial condition drift for both limiting value errors. Within a steep increase, the forecasting using the shortest drift was made more accurate because of frequent initial condition drift at the limiting value error 7 %.

In the forecasting term *October 2003 – March 2004* the initial condition drift was caused by the prognosis in the third month of the term (December 2003, the percentage prognosis error was -8,24 %). Since the price increase continued, increasing prognoses errors initialized the next initial condition drift. The advantage of lower limiting value error 7 % was the fact that the next drift occurred earlier than at the limiting value error 8 % (one-month drift: 7 %: the fourth month (-7,99 %), 8 %: the fifth month (-12,08 %); drift to the limiting month: 7 %: the fifth month (-7,27 %), 8 %: the next drift did not occur), (Figure 4, Figure 5).

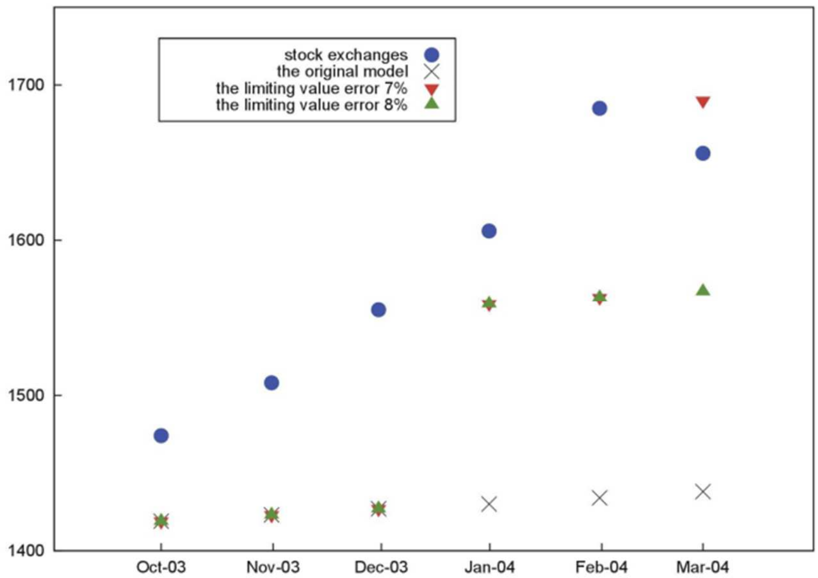

Figure 4 The forecasting by drift to the limiting month within October 2003 – March 2004 (variant B)

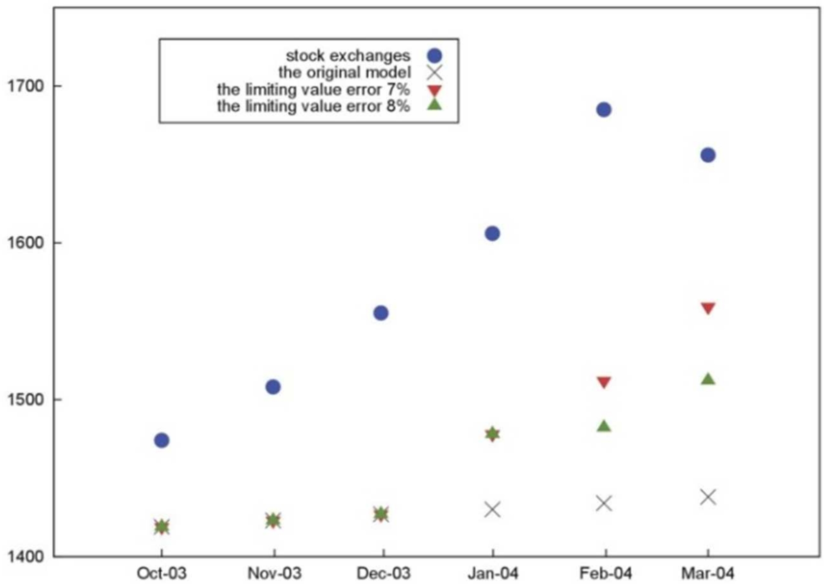

Figure 5 The forecasting by one-month drift within October 2003 – March 2004 (variant B)

Using the medium initial condition drift, the same forecasting results for both limiting value errors were acquired (Figure 6).

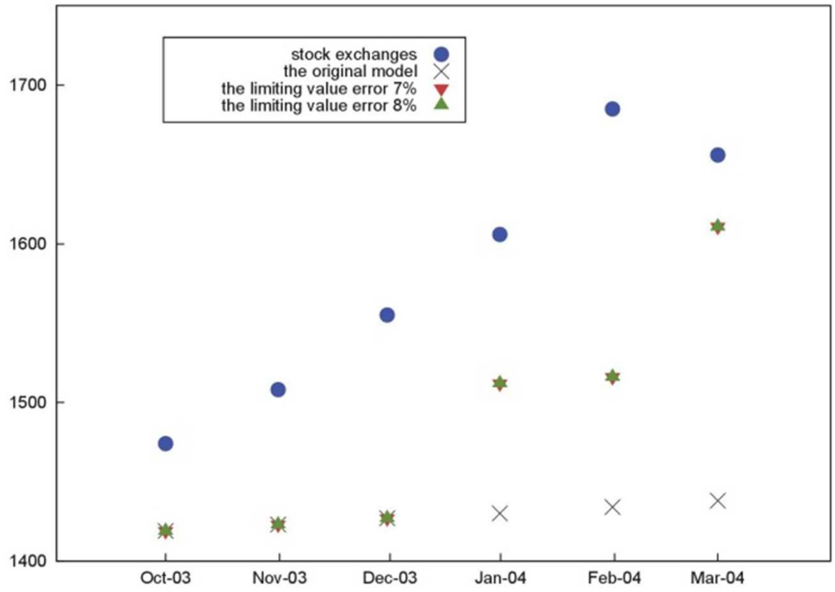

Figure 6 The forecasting by drift before the limiting month within October 2003 – March 2004 (variant B)

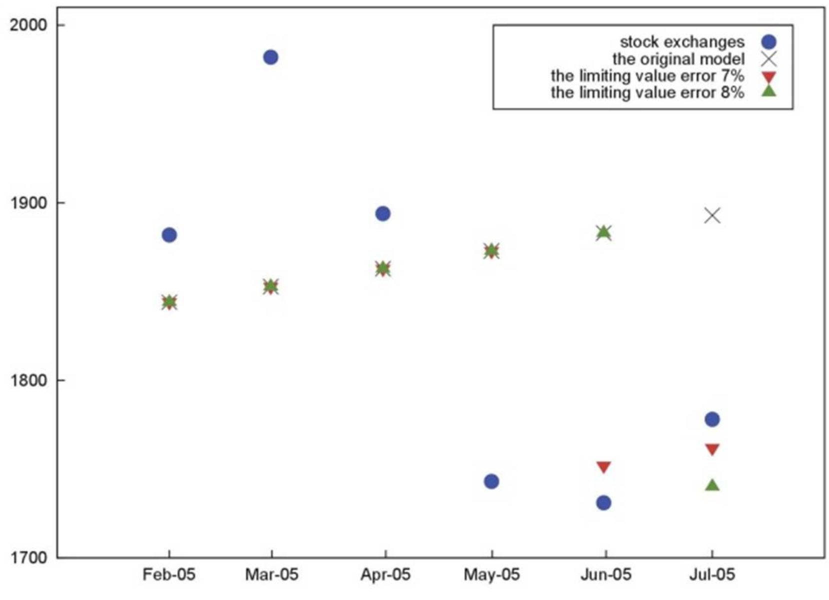

Figure 7 The forecasting by drift to the limiting month within February 2005 – July 2005 (variant E)

It was due to the fact that within price increase longer drift was more advantageous in comparison to shorter drift [5]. Thus the price prognosis in the fourth month was more accurate than when using the shortest drift (with percentage error -5,86 %). Thereby the next initial condition drift occurred not sooner than in the fifth month (the prognosis with percentage error -10,05 %). The size of the absolute percentage prognosis error caused the initialization of the drift for both limiting value errors.

- **changes in price evolution**

Within changes in the commodity price evolution the forecasting was the most problematic considering previous opposite price movement within the approximation term [4]. The prognoses with absolute percentage error ≥ 8% were most frequent, so for both chosen limiting value errors the same forecasting results were obtained. At the longest drift, the initial condition value acquired size near the price evolution, so no further drifts were usually needed in the next forecasting. The advantage of the limiting value error 7 % was occurred only if the absolute percentage prognosis error causing the initial condition drift was from the interval ⟨7%, 8%⟩. Using shorter drifts, which were less accurate within changes in price evolution, the forecasting by means of the limiting value error 8 % obtained better results.

Within the forecasting term *February 2005 – July 2005*, when there was a steep price decline after a price increase, the absolute percentage prognosis error in May 2005 was only 7,46 % (the lowest from all forecasting terms interfering with the price decline). Thus the initial condition drift at the limiting value error 7 % occurred earlier (at value 8 % the drift occurred only in June 2005). An earlier drift was advantageous only in connection with the longest drift allowing to approach immediately stock exchanges in decrease, and thus the forecasting became more accurate without further initial condition drift (Figure 7).

Within the forecasting term *June 2005 – November 2005*, when a steep price increase after a price decline occurred, the initial condition drift was caused by a prognosis in the fifth month (October 2005, percentage prognosis error -7,12 %). The drift made the forecasting more accurate only if it approached increasing values (longer drifts), (Figure 8). On the contrary, at one-month drift the initial condition value moved away from forecast steeply increasing stock exchange (it acquired the value of the aluminium stock exchange in June 2005, which was the lowest in observed forecasting term). Thus at this length of the initial condition drift the forecasting by means of the limiting value error 8 % was more successful when the initial condition drift did not occur.

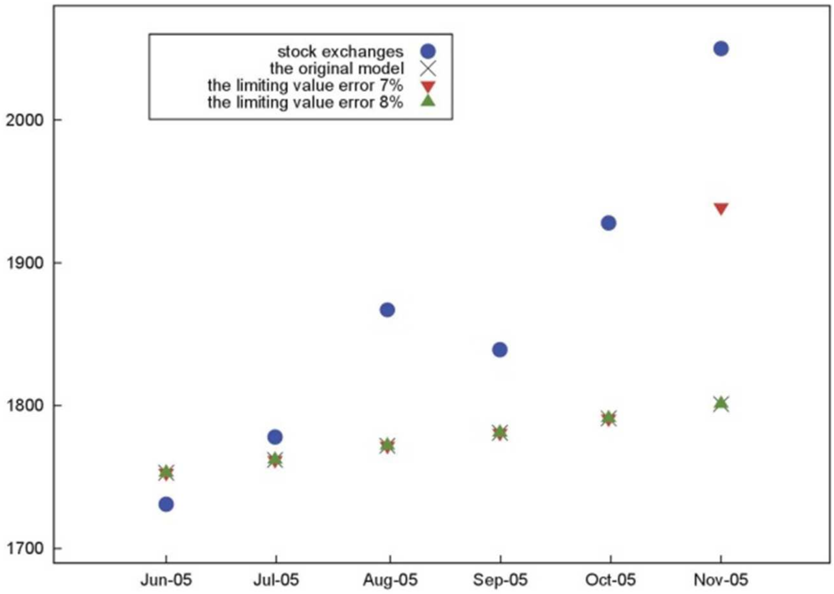

Figure 8 The forecasting by drift to the limiting month within June 2005 – November 2005 (variant E)

3.3 The forecasting success of higher limiting value error 8 % at the commodity price forecasting

The forecasting by means of higher limiting value error 8 % was more accurate than forecasting at the limiting value error 7 % only in some cases using disadvantageous lengths of the initial condition drift. If within the forecasting term we considered the most accurate length of the initial condition drift, we stated that more accurate prognoses were always obtained by means of the limiting value error 7 %. A partial success rate of a higher limiting value error was observed within significant price fluctuation and at the changes in price evolution [5].

- **significant price fluctuation**

Within significant price fluctuation higher limiting value error was advantageous only in connection with the longest initial condition drift. Within this price course the most accurate were the drifts that allowed placing the initial condition value the nearest to the real stock exchanges evolution. Using the longest drift the initial condition value was often replaced by the local maximal or minimal value that had caused the initial condition drift, which was not advantageous for forecasting following unstable price course.

Within the forecasting term *January 2006 - June 2006* higher limiting value error caused higher inaccuracy of prognoses in the third and the fourth months of the term, because the initial condition drift had occurred only at the limiting value error 7 %. But later initial condition drift in the fourth month, made it possible to capture the steep increase of the stock exchange in the fifth month (May

2006, percentage prognosis error -7,90 %). Since the next initial condition drift at the limiting value error 8 % did not occur, a suitable prognosis was acquired in the last month of the term, when the price steeply decreased, too (Figure 9).

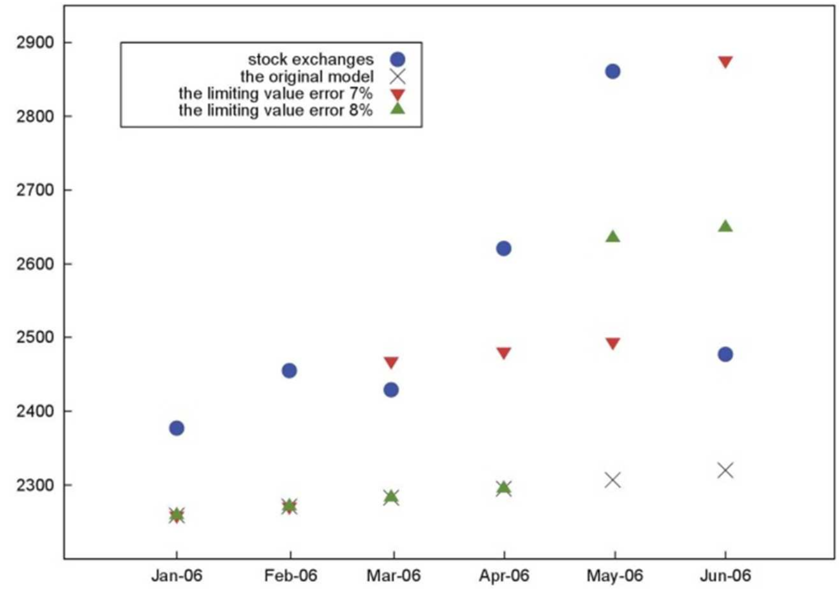

Figure 9 The forecasting by drift to the limiting month within January 2006 – June 2006 (variant E)

- **changes in price evolution**

Within a significant change in the price evolution, the advantage of the forecasting at the limiting value error 8 % was obvious only at the shorter initial condition drifts, which were least accurate during the observed price movement (Figure 10).

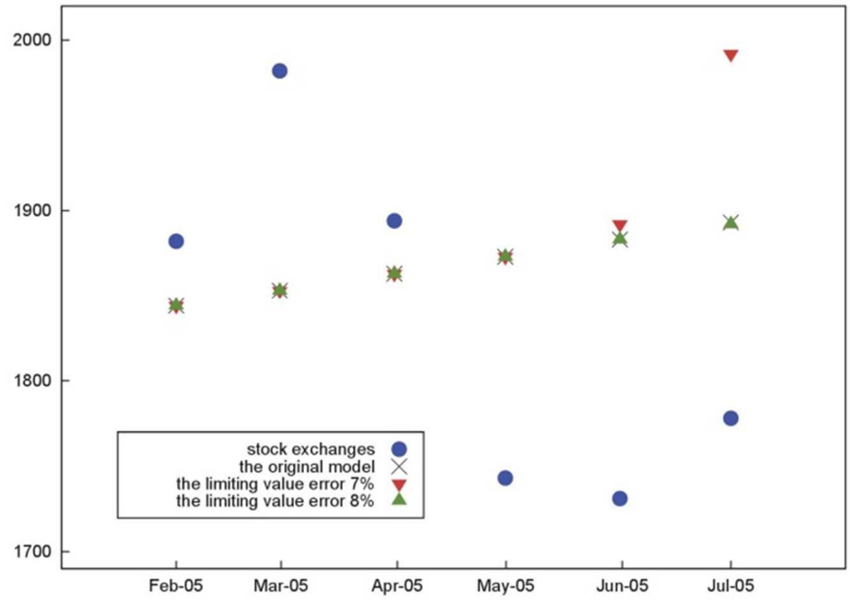

Figure 10 The forecasting by one-month drift within February 2005 – July 2005 (variant E)

By multiplying shorter drifts, the initial condition values used to remain the same as at the beginning of the forecasting term before the change. That was disadvantageous for forecasting the stock exchanges after evolution change [5]. Thereby the next prognoses often gained errors causing the initial condition drift. This situation was improved due to a lower number of drifts, provided by higher limiting value error 8 %.

Conclusions

By having analyzed obtained results, it was found out that if the forecasting results differed at the determined limiting value errors, more accurate prognoses by means of the limiting value error 7 % were more often gained. The lower limiting value error was more sensitive to prognosis error especially at a stable increase and during changes in the price course. An earlier initial condition drift usually allowed more accurate forecasting.

The limiting value error 8 % was more succesfull only in singular cases associated with certain lengths of the initial condition drift within the changes in the price course and significant price fluctuation. In all these cases within the forecasting term the most accurate prognoses were obtained while using another length of the initial condition drift in combination with the limiting value error 7 %. Based on these results, we recommend the limiting value error 7 % as more suitable for commodity price forecasting.

Acknowledgement
The research for this article was supported by KEGA 072 TUKE-4/2014.

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Review process
Single-blind peer reviewed process by two reviewers.