THE USE OF PROGRESSIVE GRAVITATIONAL METHODS IN THE LOGISTICS OF RAIL PASSENGER TRANSPORT

1 Introduction

Gravitational models and methods are generally used in activities that show certain facts and produce specific results. They are based on conceptual thinking, system approach and exact features. They are engaged in gathering materials, constantly observing and investigating different processes. Subsequently, the measurement phase consists of a phase of counting and scaling, and finally, the experimental phase in which the process under investigation is carried out in its basic conditions and at the same time is isolated from other unimportant circumstances. Within the experiment, the process is changed based on the measurements, surveyed conditions and other selected criteria. Gravitational methods belong to the empirical methods. These methods should help to generalize specific results and offer a general solution to the problem, from practical knowledge to theoretical formulation. The main instruments used in the transition to general theoretical knowledge is induction.

The widely applied application of the methods is especially in the field of transport processes and traffic planning. Basic principles of gravitational methods and models are mentioned in publication [1]. An important practical application is mostly in railway passenger transport. The methods can be used in order to calculate optimal number of journeys, return trips, number of trains and so on. Current gravitational methods and the new proposed methods can improve and optimize traffic service in railway passenger transport. Finally, it should help to make railway passenger transport more attractive and effective and to increase the quality of logistic processes.

2 Logistics and its functions in railway transport

The relationship between transport and logistics is very narrow and. Transport ensures the physical relocation of the product from the place of production to the point of consumption and logistics try to find optimal solution of the transport process. Transport is an important factor in time benefit, it is the carrier of reliability and speed of product relocation. It is one of the most substantial elements in the logistic system. It has irrecoverable and unsubstitutable place in the logistical chain from material supplier to customer.

When choosing the optimum kind of transport and the transport mode, it is very important to take into account, in
particular, the characteristics and type of the transported good, the weight, the volume, the packing methods, the legislative rules and other conditions. In this selection, account must be taken of the characteristics of the various modes of transport:

- ability to maximize transport safety, including the elimination of shocks and possible damage to the goods,
- ability to minimize the transport costs,
- the ability to provide traffic to any destination,
- ability to transport a certain number of materials and goods,
- ability to provide the optimal degree of time security and reliability,
- ability to provide the maximum degree of comfort.

Rail transport do not provide such high level of availability, manoeuvrability and flexibility as road transport, but in logistics, however its function is very important. It can provide transport in much larger quantities, in the case of freight transport offers the possibility to transport also heavy loads, shipments with specific properties, etc. Thus, it can offer a more favourable transport price per unit of consignment (e.g., ton). In the field of passenger rail transport, logistics deals with planning, organization, management and control of all activities between operators entering the transport process. The main goal is to create optimal conditions for ensuring safe, reliable, sufficiently fast and convenient passenger transport at the optimum price level. Very important is also synergy effect and cooperation with other types of transport. The basic strategic objective is to ensure the maximum quality, speed and comfort of the transport in regions and their economic development. To achieve this goal, the most appropriate solution is the introduction of an integrated transport system, integrating transport and tariff conditions, increasing the coordination of transport modes and co-operation between individual carriers. The more integrated the transport system is better for the traveling public and the logistics of public passenger transport at a higher level. The main tasks of logistics in passenger rail transport are [2]:

- review of passenger transport flows at certain times in transport routes using marketing methods or other survey methods,
- optimization and control of the transport streams,
- improving of the majority transport process factors,
- improving of the relationship with customers,
- ensuring quality services and optimal travel culture – cleanliness and comfort of the means of transport, seating, technical condition of the vehicle,
- ensuring optimal costs,
- solving the complex logistics chain – road from house to house.

3 Current research in the field of gravitational methods and models

Logistic problems in rail passenger transport and especially optimization of rail passenger transport, using various methods and models of transport planning are addressed by a large number of transport experts and scientists. Our target is focused to research a relationship between transport planning alternatively traffic service and logistics in rail passenger transport in the frame of sustainable transport system.

Using of the optimization methods Monte carlo is described in the article [3]. This method was used to optimize the fleet capacity. In [4,5] the authors focus on the quality of the service provided. As the interval between connections on individual lines is one of the qualitative indicators, such an assessment can be used. It is possible to connect innovative ideas in the field of transport using ontology. Optimisation of train traffic logistics often depends on various types of data storage and data representation. Many public transport providers are now connected to open data sets and various information systems beyond the field of rail traffic. Such domain interconnection needs smart ways of data storage for further processing and analysis. Solution described in [6] proposes using ontology as a modeling tool within the information system architecture proposal. This approach could be an inspiration in a further storage and analysis of passenger traffic data.

For example the Czech authors Jánoš and Kříž deal with issue of the gravitational methods and models in publications [7] and [8] where are explained basic principles and theoretical basis of the Lill’s gravitational models and subsequently its practical application in transport planning and determination of development forecast in Ústí nad Jímem region. Author Turner in publication [9] applies the principles of gravitational methods to air transport using several modifications and extended forms of the Lill’s gravitational model where is analyzed impact of aggregated and non-aggregated models to the issues. Within the proposal of the extended model deals with proposal of the new formula for model determination where are proposed several variables, the values of which are determined by expert estimation.

4 Current theoretical concept of the gravitational methods and models

This method is often used to calculate the direction of traffic flows in a four-stage traffic model. It is an important synthetic method that uses knowledge from another field of science (especially physics), taking into account Newton's law of gravitation, whose definition is as follows: “Consider two bodies of masses \( m_1 \) and \( m_2 \). The distance between the centers of masses is \( r \). According to the law of gravitation, the gravitational force of attraction \( F(1) \) with which the two masses \( m_1 \) and \( m_2 \) separated by a distance \( r \) attract each other is given by” [3]:

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The Use of Progressive Gravitational Methods in the Logistics of Rail Passenger Transport

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\[ F = G \times \frac{m_1 \times m_2}{r^2} \]  

(1)

After the application of the law of gravitation to the direction of transport flows, the formula (2) is follow:

\[ D_{ij} = k_{ij} \times \frac{DZ_i \times DC_j}{w_{ij}} \]  

(2)

where:

- \( D_{ij} \) - the number of journeys between areas \( i \) and \( j \) [piece],
- \( k_{ij} \) - gravitational constant [-],
- \( DZ_i \) - transport attractiveness of the area \( i \),
- \( DC_j \) - transport attractiveness of the area \( j \),
- \( w_{ij} \) - transport distance between areas \( i \) and \( j \) [km],
- \( \alpha \) - empirical value approaching 2 [-].

It is possible to take into account the more attractive the two monitoregravityd areas \((i, j)\) and the lower the transport distance between them, thus higher traffic flows can be expected and higher number of journeys between them. The mentioned problem is expressed graphically in fig. 1.

One of the most important gravitational methods is Lill’s gravitational model. The Lill’s gravitational model and its various modifications are described in other subchapters [10].

![Figure 1. Basic principles of gravitational methods [11]](image)

4.1 Basic form of the Lill’s gravitational model

Lill’s gravitational model represents the most important source of motivation and inspiration for authors in scientific research. This model was created on the basis of the research activities of the Austrian railway engineer and transport modeling expert Eduard Lill, who already in 1891 presented the main principles and laws of travel in Vienna. Given the growing importance of his ideas, it was considered appropriate to apply the principles of his travel law to rail transport. Lill considered his observations to be a natural law analogous to Newton's law of gravitation in physics, but he had a fundamentally different theoretical approach. The original formulation was based on the hypothesis that there is a certain relationship between traffic flows between certain areas (settlements), the attractiveness of these areas and their distance. These traffic flows or the number of trips (passengers) within a given area is also called the “travel value” and can be expressed as follows (3) [10]:

\[ y = \frac{M}{x} \]  

(3)

where:

- \( y \) – travel value, number of trips [piece].
- \( M \) – attractiveness of monitored areas due to their size, economic level and other characteristics,
- \( x \) – transport distance [km].

The above formulation of the relationship expresses the direct proportionality of the number of people with the attractiveness of the area and the indirect proportionality with the transport distance between them. This means that the number of passengers between two areas (settlements) decreases with increasing distance according to the hyperbolic curve 1/x. The hypothesis from which this formulation arose was subsequently confirmed and on the basis of further Lill’s research new relationships were formed expressing the given issue [10].

4.2 Current modified forms of the Lill’s gravitational model

For example, research based on determining the number of passengers leaving an area with an attraction \( M \) (i) to an area with an attraction \( M \) (j) at a distance \( x \) (j) by comparing the probability \( P \) (j-1) that passengers from area \( i \) will be transported to area \( j \) with and stop and probability \( P \) (j + 1) that passengers will be transported from area \( i \) to area \( j \) without stop. The total probability will then be expressed as their difference - \( P \) (j) = \( P \) (j-1) - \( P \) (j + 1). Subsequently, it will be possible to work on a modified relationship for determining the number of passengers going from \( i \) to \( j \), which additionally contains the variable \( L \) (j), which expresses the interval between individual connections between areas \( i \) and \( j \). The modified form of the relationship is as follows (4):

\[ y(i, j) = \frac{M(i)}{x(j) - L(j) / 2} - \frac{M(i)}{x(j) + L(j) / 2} \approx \frac{M(i) + L(j)}{x(j)^2} \]  

(4)

Another modified form of the model is based on identical principles and similar indicators as the gravitational method itself. This is the following form (5):

\[ v_{ij} = k \times \frac{Q_i \times Z_j}{w_{ij}} \]  

(5)

where:

- \( v_{ij} \) - traffic flows between areas \( i \) a \( j \),
- \( k \) – gravitational constant, [-]
- \( Q_i \) – attractiveness (potential) of the i-th starting area,
- \( Z_j \) – attractiveness (potential) of the i-th final area,
- \( w_{ij} \) – variable expressing traffic resistance.

The stated form of the model is based on the assumption that the traffic relationship between the areas \((Q_i, Z_j)\) increases with their size and attractiveness and decreases with increasing so-called traffic resistance (usually represented by travel time or transport distance between areas). The gravitational constant was determined on the basis of research and expert estimates and it expresses a certain territorial characteristic of a given area and within this model it takes values from 0.7 to 0.9. The
result of the calculation is then a numerical value, which after the transfer expresses the "relative size" of the transport flow. This form of the model was also used in the research activities of the authors Jánoš and KFH in their publications focused on transport planning in regional rail passenger transport.

Another modification differs in addition to the different designations of the variables in the numerator of the relationship, but their meaning is not changed. Denominator variable expresses traffic resistance through the transport distance between areas i and j (lij), which is raised to the second power. This form of the formula is as follows (6):

$$v_{ij} = k \times \frac{z_i \times c_j}{(lij)^2}$$  \hspace{1cm} (6)

An alternative to the above modification is to express the transport resistance through the general transport costs (cij) instead of the transport distance (7):

$$v_{ij} = k \times \frac{z_i \times c_j}{(cij)^2}$$  \hspace{1cm} (7)

The best known and the most widespread modification of the Lill model, which is currently the most widely used, is the form used to determine the optimal number of return journeys of all public passenger transport modes between two selected transport points. It takes into account the population of these transport points and the distance between them. The calculated optimal number of journeys (8) is directly proportional to the number of inhabitants of both transport points, the gravitational coefficient K and indirectly proportional to the distance of these transport points. The model has the following shape, the result is always rounded up [10]:

$$j_{1,2} = \frac{A_1 \times A_2}{\pi} \times K$$  \hspace{1cm} (8)

where:

- $j_{1,2}$ - optimal number of return journeys (connections) between chosen traffic points,
- $A_1, A_2$ – number of inhabitants (in thousands) of the traffic points,
- d – transport distance between traffic points [km],
- K – gravitational constant (depends on the nature and connection of the selected settlements),
- n – variable approaching 2.

### 4.3 New progressive form of the Lill’s gravitational model

Gravitational models are the inspiration for the creation of the following relationship, which can be considered as a new progressive gravitational method applicable in regional rail transport. The resulting form of the proposed formula for the calculation of the transport potential ($K_p$) can be marked as a further modification of Lill’s gravitational model. Its form is as follows (9) [12]:

$$K_p = \frac{\sum n \times A_n}{L}$$  \hspace{1cm} (9)

where:

- $K_p$ – traffic potential coefficient [population/km²],
- $A_n$ – the number of inhabitants of the n-th seat of the monitored area [piece],
- $D_n$ – availability of the n-th railway station and stop – its distance from the centre or from its middle [piece],
- L – length of the railway passenger transport route [km].

The most significant benefit of the proposed relationship is the assessment of traffic potential and subsequent more efficient identification of bottlenecks on individual regional rail routes, while it is possible to assess whether the line has low transport potential due to low population density or the problem is accessibility of railway stations, and their longer distance from settlements. It is also possible to assess the effectiveness of investments in railway infrastructure on these lines, whether, based on the transport potential, more extensive construction and reconstruction measures will pay off, or operational and organizational measures will be sufficient, etc. Based on the calculated values of $K_p$ on several transport routes, it is possible to better compare these routes and then assess their importance on the basis of the resulting hierarchy and thus the prioritization of possible modernization. Based on scientific opinions and brainstorming methods, the width of the intervals is as follows [12,13].

### Table 1 Determination of the modified intervals width of particular traffic service ranges [12]

<table>
<thead>
<tr>
<th>Traffic service range</th>
<th>The interval of the resulting $K_p$ value</th>
<th>Recommended number of pairs of all regional trains</th>
<th>Recommended number of seats for all train connections in both directions</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>0 – 700</td>
<td>4</td>
<td>up to 500 seats</td>
</tr>
<tr>
<td>II.</td>
<td>701 – 1 000</td>
<td>5 – 6</td>
<td>250 – 1 500</td>
</tr>
<tr>
<td>III.</td>
<td>1 001 – 1 200</td>
<td>7 – 10</td>
<td>350 – 3 000</td>
</tr>
<tr>
<td>IV.</td>
<td>1 201 – 1 400</td>
<td>11 – 15</td>
<td>550 – 6 000</td>
</tr>
<tr>
<td>V.</td>
<td>1 401 – 1 600</td>
<td>16 – 20</td>
<td>1 000 – 8 000</td>
</tr>
<tr>
<td>VI.</td>
<td>1 601 – 1 800</td>
<td>21 – 25</td>
<td>2 000 – 10 000</td>
</tr>
<tr>
<td>VII.</td>
<td>1 801 – 2 000</td>
<td>26 – 30</td>
<td>4 000 – 12 000</td>
</tr>
<tr>
<td>VIII.</td>
<td>2 001 – 2 300</td>
<td>31 – 39</td>
<td>5 000 – 15 000</td>
</tr>
<tr>
<td>IX.</td>
<td>2 501 – 3 000</td>
<td>40 – 49</td>
<td>7 000 – 20 000</td>
</tr>
<tr>
<td>X.</td>
<td>3 001 and more</td>
<td>50 and more</td>
<td>8 000 and more</td>
</tr>
</tbody>
</table>

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5 Practical application of the new progressive Lill’s gravitational model

The 3rd chapter contains a practical application of the new progressive Lill’s gravitational model. On the table 2 and table 3 are four model Slovak railway transport routes (Bratislava–Trnava, Žilina–Čadca, Púchov–Horní Lideč, Plešivec–Slavošovce).

<table>
<thead>
<tr>
<th>Table 2 Kp calculation on Slovak railway transport routes Bratislava – Trnava and Žilina - Čadca</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kp calculation on Bratislava–Trnava railway line</strong></td>
</tr>
<tr>
<td><strong>Tariff point:</strong></td>
</tr>
<tr>
<td>Svätý Jur</td>
</tr>
<tr>
<td>Pezinok</td>
</tr>
<tr>
<td>Šenkvice</td>
</tr>
<tr>
<td>Báhoň</td>
</tr>
<tr>
<td>Cifer</td>
</tr>
<tr>
<td>Trnava</td>
</tr>
<tr>
<td><strong>Total value (A/D)</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3 Kp calculation on Slovak railway transport routes Púchov – Horní Lideč and Plešivec – Slavošovce</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kp calculation on Púchov–Horní Lideč line</strong></td>
</tr>
<tr>
<td><strong>Tariff point:</strong></td>
</tr>
<tr>
<td>Dohňany</td>
</tr>
<tr>
<td>Záriečie</td>
</tr>
<tr>
<td>Mestečko</td>
</tr>
<tr>
<td>Lúky p. M.</td>
</tr>
<tr>
<td>Lysá p. M.</td>
</tr>
<tr>
<td>Strelenka</td>
</tr>
<tr>
<td>Střelná</td>
</tr>
<tr>
<td>Horní Lideč</td>
</tr>
<tr>
<td><strong>Total value (A/D)</strong></td>
</tr>
<tr>
<td><strong>Final Kp value</strong></td>
</tr>
</tbody>
</table>

Within each specified transport route, the traffic potential is calculated in particular tariff points according to the formula stated in subchapter 2.3. Each tariff point includes the population (A), distance of the railway stop from the town/village centre (D) and finally partial and final value of traffic potential (Kp).

Final Kp values including optimal traffic service range (according to table 1) compared with actual traffic service range is situated in table 3. It follows from the calculated data that optimal traffic service range is IX (40 – 49 pairs of the regional trains during working day) and actual traffic service range in 2020 is VII (35 pairs) on the railway transport route Bratislava – Trnava. It follows that actual traffic service is not sufficient and it is necessary to increase it. Optimal traffic service range on Žilina – Čadca railway line is VII (26 – 30 pairs of the regional trains during working day) and actual traffic service range in 2020 is V (20 pairs). It also follows that it is appropriate to introduce 6 - 10 new pairs of passenger trains. Optimal traffic service range on the railway transport route Púchov – Horní Lideč is II (5-6 pairs of regional trains during working day) and actual traffic service range in 2020 is III (7 pairs). It follows that it is appropriate to introduce 6 new pairs of passenger trains to meet the basic transport needs of passengers. And finally optimal traffic service is I (4 pairs of regional trains during working day) on the transport routes Plešivec – Slavošovce, but at present there...
is no railway passenger trains. There must be introduced a minimum of 4 pairs of regional passenger trains.

<table>
<thead>
<tr>
<th>Railway transport route</th>
<th>Final Kp value</th>
<th>Optimal traffic service range</th>
<th>Actual traffic service range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bratislava–Trnava</td>
<td>2501.94</td>
<td>IX</td>
<td>VII</td>
</tr>
<tr>
<td>Žilina–Čadca</td>
<td>1341.99</td>
<td>VII</td>
<td>V</td>
</tr>
<tr>
<td>Púchov–Horní Lideč</td>
<td>815.21</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>Plešivec–Slavošovce</td>
<td>616.03</td>
<td>I</td>
<td></td>
</tr>
</tbody>
</table>

6 Conclusion

The current gravitational models used in transport processes, especially in railway passenger transport, are analyzed in the article. The Authors have described and analyzed particular modifications of Lill’s gravitational model. Subsequently, a new proposed modification was introduced. Finally, the practical application of the new model was presented. Within it the authors have calculated traffic potential (Kp) at five chosen railway transport routes.

Based on the above-mentioned more objective assessment, it would also be possible to estimate more accurately the passenger traffic flows, define bottlenecks of selected transport routes. On this basis, there is a prerequisite for improving the train traffic diagram, linking the different types of transport, increasing the number of passengers, increasing customer satisfaction, as well as a comprehensive improvement and improvement in rail transport that has stagnated in recent years. From the point of view of logistics, the gravitational models and methods should help to find the optimal alternative for moving people with the maximum synergy effect. The main result should be a high-quality logistical chain with satisfied customers [14,15].

Passenger transport logistics can also be solved using other algorithms, as described, for example, in the article [16]. Although the article is focused on the transport of cars, it is worth using the algorithm proposed here also on the problem of passenger transport logistic.

Equally important is the transport of immobile passengers. This is discussed in the article [17]. Within the number of passengers and the number of trains, this would be a restrictive condition in our relationship. On this basis, however, it would be possible to determine the number of barrier-free connections.

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