

FLEET OPTIMIZATION BASED ON THE MONTE CARLO ALGORITHM

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Abstract: With the development of computers and software products, there is now greater use of quantitative methods in industrial enterprises when making managerial decisions. One of the most applicable solutions to computer simulation algorithms is the Monte Carlo method. The application of the Monte Carlo algorithm lies in finding a relation between the individual variables, which are the solutions to the problem and represent the characteristics of random processes reproducible on computers. The aim of this article is to show the application of simulations from the Monte Carlo algorithm using the example of optimising vehicle fleet capacity so that the total daily costs spent on transporting goods are minimal.

1 Introduction

Over the last two decades, there have been great advances in computer technology and these advances have also affected managerial decision-making [1]. Mathematical methods with the use of computer technologies have been increasingly used in most problems of managerial decision-making [2,3]. One of the most applicable methods of computer implementation is the Monte Carlo method [4]. A static, stochastic method uses random (pseudo-random) numbers in the course of the calculation [5]. In this article, its use will be shown while optimising vehicle fleet capacity so that the daily costs spent on transporting goods are minimal.

2. Monte Carlo simulation

The term Monte Carlo is widely used to denote a wide class of computing methods that use random sampling to obtain numerical solutions. Monte Carlo methods are ubiquitous in science and engineering; they are preferred due to their simplicity [6].

Monte Carlo methods are used to simulate the behaviour of physical or mathematical systems, especially when analytical solutions are difficult to obtain [7]. These methods are nondeterministic or stochastic. The applications of Monte Carlo methods are very diverse: these include physics, computer science, engineering, environmental sciences, finance, fleet management etc. And systems with uncertainties other than pure mathematical systems that require no uncertainty [8].

A random value of x is known as a random number. In practice, x values can be determined deterministically, and the random numbers so generated are known as pseudo-random numbers: such a pseudo-random number contains a limited number of digits, which means that the continuous uniform distribution is approximated by the discrete. Pseudorandom numbers are often used in

simulation studies [9]. Monte Carlo methods require many pseudo-random numbers for computer processing.

Simulation models are usually used to decide on something that involves a risk, i.e. a model in which the behaviour of one or more components is not known with certainty. Bank lines or the number of trucks that arrive between specific time intervals are cases where a component that is not listed with confidence is considered a random variable and a probability distribution is used to simulate the behaviour of the random variable [10].

In the practical example used, the simulation is applied to the number of customer orders per day, operator intensity, and the simulated expected overtime operation.

3. The task

A company provides the transportation of goods based on customer orders according to the following rule:

Table 1 Day of ordering and the subsequent transport of the goods

Ordered on	Transported on
Monday	Tuesday
Tuesday	Wednesday
Wednesday	Thursday
Thursday	Friday
Friday	Monday

Orders are not received on Saturdays or Sundays and there is no transportation on these days. Transportation is carried out by using trucks. The daily cost of operating one truck is 600 € – provided the trucks operate 8 hours each day.

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If an order requires overtime operation, i.e. an operation that exceeds the set 8-hour operational time, the costs for each overtime hour are 250 €.

Through statistical investigation for the assumption of a normal distribution of probability, the following data were determined:

- Data on the transport capacity of cars – on average, during an 8-hour running time, 1 car transports 80 orders with a standard deviation of 15 orders,
- Data on customer demand.

Table 2 Individual customer demands per respective days

Days	The average number of orders x	Standard deviation σ
Monday	7,800	800
Tuesday	6,400	700
Wednesday	5,500	600
Thursday	7,000	750
Friday	5,000	500

The business unit must set an optimal vehicle fleet capacity so that the total daily costs spent on transportation are minimal.

4. Analysis of the solution

From the substantive point of view, this is a task that belongs to the methodology of the queuing theory [11]. It is a simple task to determine the optimal dimension (size, capacity) of the operating system:

- Operating system = vehicle fleet that provides transportation services,
- Two main variables are assumed (variables crucial to solution of the task)
 - operation intensity = transport capacity of the vehicle,
 - the intensity of the requirements entering the operating system = the number of orders.
- Number of the contracts has the character of random variables with a normal distribution of probability.

Two extreme situations (two extreme variations in the capacity of the operating system) that lead to relatively high costs of the activity may occur in terms of optimisation:

- A heavily oversized operating system (too many vehicles):
 - high costs of the vehicle fleet,
 - costs of overtime work converge to zero; the requests do not queue.
- A heavily undersized operating system (the number of vehicles is too small):
 - low costs of the vehicle fleet,

- high costs for overtime work; formation of request queues (possible loss of profit due to queues).

Between the extreme variants, there is a number of possible variants of dimensioning the vehicle fleet that are associated with the different daily costs of transportation [12,13]. Of the possible variants, it is necessary to choose the optimal variant - the one with the lowest daily costs for transportation.

The following applies (1) for the i -th variation of the operating system dimension:

$$N = N_V + N_M \tag{1}$$

N – daily costs for transportation,
 N_V – daily costs for operating the vehicle fleet,
 N_M – daily overtime cost.

5. The procedure of solving the task using a simulation of the Monte Carlo algorithm

For the assumption of normal probability distribution of both main variables:

1. The simulations (predictions) of the number of orders are performed according to the intensity of the requirements entering the operation. The simulation itself is in Table 3.

Table 3 Simulation of the number of orders

Day	Simulated value of the distribution function ^{*)}	Determinant variable t	Prediction ^{**)} of the number of contracts $x = \bar{x} + t \cdot \sigma$
Mon	0.88147	1.1	8,680
Tue	0.9970	2.7	8,290
Wed	0.11941	-1.1	4,840
Thu	0.78817	0.8	7,600
Fri	0.17061	-0.9	4,550

*) from the table of random numbers, we assume five-digit values for the distribution function of the normal probability distribution.

***) Prediction of the number of orders (2):

$$x = \bar{x} + t \cdot \sigma \tag{2}$$

For Monday: $x = 7,800 + 1.1 \times 800$ **$x = 8,680$**
 For Tuesday: $x = 6,400 + 2.7 \times 700$ **$x = 8,290$**
 For Wednesday: $x = 5,500 - 1.1 \times 600$ **$x = 4,840$**
 For Thursday: $x = 7,000 + 0.8 \times 750$ **$x = 7,600$**
 For Friday: $x = 5,000 - 0.9 \times 500$ **$x = 4,550$**

\bar{x} – the average number of orders,
 σ – determinant deviation.

2. A simulation of the operational intensity of the 8-hour transportation capacity for the chosen number of vehicles is carried out: 70, 80, ..., 110

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n – the number of vehicles,
 \bar{x} – mean value of operational intensity,
 \bar{x}' – the average transport capacity of the fleet.

Determining the number of vehicles for the highest number of orders:
 The highest number of orders = 7,800.
 Determinant deviation = 800.
 One vehicle transports an average of 80 orders per hour.
 The total number of vehicles for the highest number of orders = $(7,800 + 800)/80 = 110$.

Determination of the number of vehicles for the lowest number of orders:

The lowest number of orders = 5,000.
 Determinant deviation = 500.
 One vehicle transports 80 orders per hour.
 The total number of vehicles for the lowest number of orders = $(5,000 + 500)/80 = 70$.

In the following step (Table 4), a simulation of the operational intensity is performed.

Table 4 Simulation of operational intensity

The Number of vehicles n	Day	The simulated value of the distribution function of normal distribution	Determinant variable t	$\bar{x}' = n \cdot \bar{x}$	$\dot{\sigma} = \sqrt{n} \cdot \sigma$	Prediction of 8-hour transport capacity $\bar{x}' = x + t \cdot \dot{\sigma}$
70	Mon	0.33166	-0.4	5,600	126	5,550
	Tue	0.87094	1.1	5,600	126	5,739
	Wed	0.11120	-1.2	5,600	126	5,449
	Thu	0.22254	-0.7	5,600	126	5,512
	Fri	0.96023	1.7	5,600	126	5,814
80	Mon	0.76869	0.7	6,400	134	6,494
	Tue	0.39300	-0.2	6,400	134	6,373
	Wed	0.02982	-1.8	6,400	134	6,159
	Thu	0.57991	0.2	6,400	134	6,427
	Fri	0.94479	1.6	6,400	134	6,614
90	Mon	0.96023	1.7	7,200	142	7,441
	Tue	0.88936	1.2	7,200	142	7,370
	Wed	0.88936	-0.5	7,200	142	7,129
	Thu	0.55013	0.1	7,200	142	7,214
	Fri	0.10920	-1.2	7,200	142	7,030
100	Mon	0.26299	-0.6	8,000	150	7,910
	Tue	0.77806	0.7	8,000	150	8,105
	Wed	0.12446	-1.1	8,000	150	7,835
	Thu	0.23510	-0.7	8,000	150	7,895
	Fri	0.68774	0.4	8,000	150	8,060
110	Mon	0.48454	0	8,800	157	8,800
	Tue	0.65269	0.4	8,800	157	8,863
	Wed	0.18167	-0.9	8,800	157	8,659
	Thu	0.84631	1.0	8,800	157	8,957
	Fri	0.74108	0.6	8,800	157	8,894

Calculation example for $n = 70$
 $\bar{x}' = 70 \times 80 = 5.600$
 $\dot{\sigma} = \sqrt{70} \cdot 15 = 126$

3. The expected overtime is simulated (from the simulation of the number of orders and the eight-hour capacity of the fleet) in Table 5.

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Table 5 Simulation of expected overtime operation

n	Day	Simulation		Prediction of expected overtime operation			
		The number of orders	8-hour capacity of the vehicle fleet	The number of pending orders	The number of overtime hours	The total number of overtime hours	The average number of daily overtime hours
70	Mon	7,800	5,550	2,250	225	445	89
	Tue	6,400	5,739	661	66		
	Wed	5,500	5,449	51	5		
	Thu	7,000	5,512	1,488	149		
	Fri	5,000	5,814	0	0		
80	Mon	7,800	6,494	1,306	131	191	38.2
	Tue	6,400	6,373	27	3		
	Wed	5,500	6,159	0	0		
	Thu	7,000	6,427	573	57		
	Fri	5,000	6,614	0	0		
90	Mon	7,800	7,441	359	36	36	7.2
	Tue	6,400	7,370	0	0		
	Wed	5,500	7,129	0	0		
	Thu	7,000	7,214	0	0		
	Fri	5,000	7,030	0	0		
100	Mon	7,800	7,910	0	0	0	0
	Tue	6,400	8,105	0	0		
	Wed	5,500	7,835	0	0		
	Thu	7,000	7,895	0	0		
	Fri	5,000	8,060	0	0		
110	Mon	7,800	8,800	0	0	0	0
	Tue	6,400	8,863	0	0		
	Wed	5,500	8,659	0	0		
	Thu	7,000	8,957	0	0		
	Fri	5,000	8,893	0	0		

6. The result of the task - recommended by the simulation calculations

A calculation of the total daily costs for transportation with the individual numbers of vehicles is shown below:

$$N = N_V + N_M$$

$$N_{70} = 70 \times 600 + 89 \times 250 = 64,250 \text{ €}$$

$$N_{80} = 80 \times 600 + 38.2 \times 250 = \mathbf{57,550 \text{ €}}$$

$$N_{90} = 90 \times 600 + 7.2 \times 250 = \mathbf{58,500 \text{ €}}$$

$$N_{100} = 100 \times 600 + 0 \times 250 = 60,000 \text{ €}$$

$$N_{110} = 110 \times 600 + 0 \times 250 = 66,000 \text{ €}$$

Under the given conditions, the total daily transport costs will be minimised when operating 80 vehicles or 80 to 90 vehicles.

7. Conclusions

As seen from the article, in this case, the use of the Monte Carlo algorithm is very appropriate when dealing with the problem of optimal vehicle fleet capacity. It was found that the overall lowest daily transport costs are 57,500 € when using 80 trucks. In this case, MS Excel was used to generate pseudorandom numbers. Computers play

an indispensable role at the application of simulations and not using them in today's field of quantitative methods of managerial decision-making is unimaginable.

Further research into fleet issues should focus on the use of telematics, which presents new possibilities to accelerate the processing of information throughout the process and increase customer satisfaction. Research trends consist in developing new efficient fleet management models, optimization and simulation models, and integrating them into the entire decision support system. Computational efficiency of methods can be increased by integrating precise algorithms and developing mechanisms based on mathematical programming principles. The use of artificial intelligence principles and methods is also the right direction.

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