doi:10.22306/al.v6i1.113
Received: 13 Feb. 2019
Accepted: 04 Mar. 2019

IN THE ISSUES OF MATHEMATICAL MODELLING LOGISTICS PROCESSES

Alexander Lozhkin
Department of Software, Institute of Informatics and Computer Engineering, Kalashnikov Izhevsk State Technical University, 7 Studencheskaya St., 426069 Izhevsk, Russia, lag.izh@gmail.com

Keywords: form of characteristic equation, solution basis, symmetries
Abstract: Mathematical modelling came to logistics from economics. Economic models have been used for quite some time, but the results obtained force us to develop new branches of the theory. The solution of the characteristic equation is the basis of these algorithms as a rule. The classical solution of the characteristic equation from geometry is used. The solution is based on two postulates: the type of the characteristic equation is not changed and the solution is obtained in an orthogonal basis. The transformation matrix change the form of the characteristic equation is proved. The symmetries of space vary it. Solutions for complex non-linear processes should be considered in a non-orthogonal basis. This basis is primary. The orthogonal basis appears from it or in a particular case.

1 Introduction

Mathematical modelling takes one of the most important stages in logistic processes. Calculations of various types of resources taken by logistics from the classical economy. Economic theory has been using mathematical modelling for quite some time [1]. The theory of optimal control [2], optimization methods, functional analysis [3] is used in solving management problems everywhere. Modern modelling methods, such as tropical mathematics [4], were developed on the basis of classical works.

From the point of view of mathematics, the main task of economic calculation is the solution of the characteristic equation $\lambda\vec{v} = \mathbf{T}\vec{v}$, where $\vec{v}$ – a certain vector is in multidimensional space, a $\mathbf{T}$ – transformation matrix, a λ – set of scalars determining the solution of the problem. Each element of the vector $\vec{v}$ belongs to one mathematical set. Widely used set of integers to simplify the task. The works of Kantorovich limit the space to certain restrictions [5,6]. Tropical mathematics imposes additional restrictions, for example, on the used operations [7].

The correct solutions are not always obtained, which forces us to develop new methods of mathematical modelling. The solution of the problem, as a rule, comes from two postulates: the form of the characteristic equation is not changed, the solution is in the orthogonal basis. Consider how these postulates are implemented in reality.

2 Characteristic equation

The solution of the characteristic equation is based on the Cartesian method [8]. It has such postulates as outlined in Chapter 1. The method is used in geometry for conic sections on Euclidean plane (space) only. More complex curves are processed on approximation exclusively.

An alternative plane representation model has been proposed [9]. He suggested that the construction of linguistic rules must be considered additionally. We obtain the Euclidean plane as a text by Galilee definition of the nature according to Leibniz method of similarity. As the basic postulates were used: permutation, mirror, and unitary matrix symmetries by Dieudonne; table automorphisms and transfer symmetry by H. Weyl; definition of symmetry by M. Born; and binary automorphisms by F. Bachman [10].

Isaac Newton proposed the first classification of plane curves [11]. To the first class he assigned conical sections with the equation $Ax^2 + 2Bxy + Cy^2 + Dx + Ey + F = 0$ The method of obtaining the characteristic equation $\mathbf{T}\vec{v} = \lambda\vec{v}$ for them was found, but it took years and studies by Klein to obtain solutions for the parabola [8]. The second class of curves is determined by the general equation $Ax^3 + 3Bx^2y + 3Cxy^2 + Dy^3 + 3Ex^2 + 6Fxy + 3Gy^2 + 3Hx + 3Ky + L = 0$. The method of solving this equation has not been found so far. Newton's classification has higher grades. We briefly described the classification for analytic geometry. Algebraic and differential geometry studies differentiable curves at the present time. We apply its methods, and also such a property of the Euclidean plane as symmetry [9].

Let there be an arbitrary figure Φ -- planar differentiable curve in the Euclidean plane $R \times R$ in a Cartesian coordinate linear system defined by the parametric equation: $\begin{cases} x = f_x(t) \\ y = f_y(t) \end{cases}$, where $x, y, t \in R$. Functions $f_x(t)$ and $f_y(t)$ are sine and cosine, where $t = n\tau$, $n \in Z$, $\tau \in R$. We carry out any transformation of the figure Φ defined by the matrix $\begin{pmatrix} a & h \\ g & b \end{pmatrix}$, where $a, b, h, g \in R$. It is necessary to obtain the parameters of the transformed figure (to solve the characteristic equation).

The permutation symmetry $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is defined on the Euclidean plane, so for a own orthogonal basis $0e_1e_2$ there must exist a symmetric frame $0e_1'e_2'$ with respect relation to the straight line AB (Figure 1). The basis $0e_1'e_2'$ is orthogonal, but the set of reference frames gives four non-orthogonal bases: $0e_1e_1'$, $0e_1e_2'$, $0e_2e_1'$, $0e_2e_2'$. Particularly interesting are the first two, since it is the vector that determines the angle of the figure's tilt. The hypothesis of preserving the balance of symmetries [9], as a syntactic rule for constructing the Euclidean plane, leads to the appearance of a local basis.

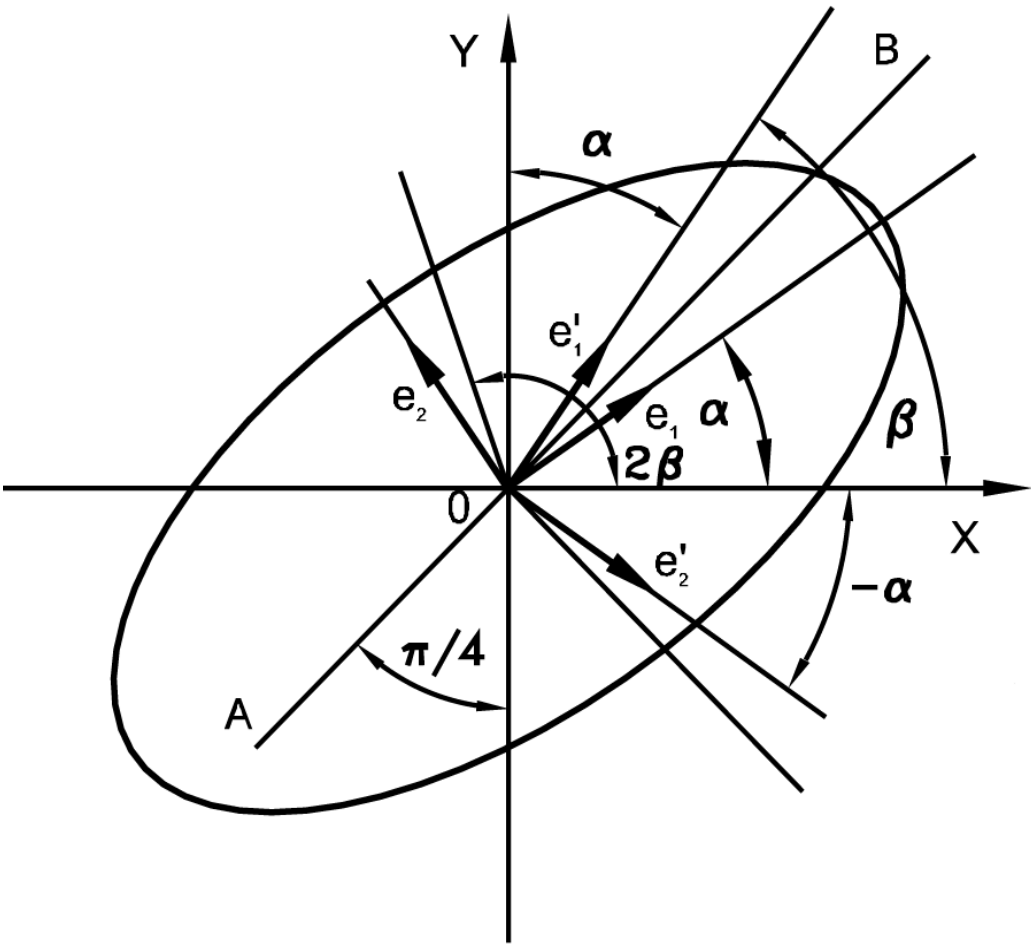

Figure 1 Non orthogonal basis

The basis defined the direct method of linear transformations. We may formulate next system of parametric equations, namely:

$$\begin{cases} cf_x(t+\alpha) = af_x(t+\beta) + hf_y(t+\beta) \\ df_y(t+\alpha) = gf_x(t+\beta) + bf_y(t+\beta)' \end{cases} \quad (1)$$

where β -- angle of permutation symmetry. Non orthogonal basis [12] is form from the corners α and β. Let consider the solution of the characteristic equation for the centrally symmetric conic sections. Only equation of own angle α take of the classic method [13,14]: $\tan 2\alpha = \dfrac{2(bh+ag)}{(a^2+h^2)-(b^2+g^2)}$. Parameters semiaxes considered difficult in the classical method, since they represent a radical dependence.

A new direct analytical method for the linear transformation was proposed earlier. He is free from radicals, so it is more simple and accessible for further mathematical derivations. The method is based on the permutation symmetry and other symmetries [8].

The general solution for planar curves has been sought for a long time. If the result of the calculations obtained systems: $\begin{cases} x = mf_x + nf_y \\ y = mf_x - nf_y \end{cases}$, $\begin{cases} x = -mf_x + nf_y \\ y = mf_x + nf_y \end{cases}$, $\begin{cases} x = mf_x + knf_y \\ y = -nf_x + kmf_y \end{cases}$, $\begin{cases} x = kmf_x + nf_y \\ y = -knf_x + mf_y \end{cases}$ than solution may be find [15]. The angle β in the basis is the defining angle. The solution is obtained when $\beta \in \{0, \pm\pi/2, \pm\pi\}$.

Theorem.

An arbitrary non-singular transformation $\mathbf{T}$ can change the characteristic equation $\mathbf{T}\begin{pmatrix} f_x(t) \\ f_y(t) \end{pmatrix} = \begin{pmatrix} \lambda_x f_x(t) \\ \lambda_y f_y(t) \end{pmatrix}$ to an equation $\mathbf{T}\begin{pmatrix} f_x(t) \\ f_y(t) \end{pmatrix} = \begin{pmatrix} \lambda_x f_y(t) \\ \lambda_y f_x(t) \end{pmatrix}$.

Proof.

Let us suppose the opposite. Let us assume an transformation $\mathbf{T}$ is not exist. Let us be a curve with parametrical equations system $\begin{cases} x = f_x(t) \\ y = f_y(t) \end{cases}$ is satisfying the initial conditions of this publication. Let us apply a linear transformation $\begin{pmatrix} 0 & k \\ n & 0 \end{pmatrix}$ to it, where $n, k \neq 0$, then the resulting trivial solution of the characteristic equation will be equal $\begin{cases} x' = kf_y(t) \\ y' = nf_x(t) \end{cases}$. The conversion $\mathbf{T}$ is not singular, since $\det \mathbf{T} = -nk \neq 0$. The equation changed its appearance.

Thus, we obtain that the characteristic equation has the form $\lambda \vec{v}' = \mathbf{T}\vec{v}$, where $\vec{v}' \in \left\{ \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} -x \\ -y \end{pmatrix}, \begin{pmatrix} y \\ x \end{pmatrix}, \begin{pmatrix} -y \\ -x \end{pmatrix} \right\}$. Not considered members of the set are obtained from the mirror and permutation symmetry.

Analytical solution found for the following types of transformation systems:

$$\begin{cases} x = f_x + nf_y \\ y = f_y \end{cases}, \begin{cases} x = f_x \\ y = nf_x + f_y \end{cases}, \begin{cases} x = f_x + nf_y \\ y = kf_x + f_y \end{cases}.$$

The solution algorithm is divided into three steps:

1. We calculate the transformation parameters as in the case of centrally symmetric conic sections.

2. We perform an additional transformation $\mathbf{P}$ depending on the angle β. If the angle β is negative, then the transformation is $\mathbf{P} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \cos\alpha & \sin\alpha \end{pmatrix}$, if positive, then $\mathbf{P} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \cos\alpha & \sin\alpha \end{pmatrix}$.

3. Rotate the curve by its own angle α and multiply it by the coefficients founded in item 1.

3 Conclusions

The own angle determines the orthogonal basis. This is the principle of the classical method for solving the characteristic equation. The angle of the permutation symmetry determines its own angle. These two angles form a non-orthogonal basis. This basis and symmetry angle is primary. This all follows from the proposed method. The characteristic equation has a more complicated form than generally accepted.

This article does not correct the mathematical model used in economics and logistics. It provides only the contours of future decisions. The author of the publication, although he was educated as an economist engineer, does not consider himself the right to make changes to the existing theory.

References

[1] LEONTIEF, WASSILY W.: An Alternative to Aggregation in Input-Output Analysis and National Accounts, *Review of Economics and Statistics*, Vol. 49 No. 3, pp. 412-419, 1967.

[2] KANTOROVICH, L.V.: *The Best Use of Economic Resources*, New York, Pergamon Press, 1965.

[3] LEIFMAN, LEV J.: (ed.) *Functional analysis, optimization, and mathematical economics: A collection of papers dedicated to the memory of Leonid Vital'evich Kantorovich*, New York, The Clarendon Press, Oxford University Press, 1990.

[4] SPEYER, D., STURMFELS, B.: Tropical mathematics, *Mathematics Magazine*, Vol. 82 No.3, pp. 163-173, 2009.

[5] CUNINGHAME, G., RAYMOND, A.: *Minimax algebra, Lecture Notes in Economics and Mathematical Sciences*, New York, Springer, 1979.

[6] HOPP, W., SPEARMAN, M.: *Factory Physics*, New York, Waveland Press, 2011.

[7] FITZSIMMONS, J, FITZSIMMONS, M. BORDOLOI, S.: *Service Management: Operations, Strategy and Technology*, 8th ed., New York, McGraw-Hill, 2014.

[8] EFIMOV, N.V.: *Quadratic forms and matrices*, Science, Moscow, 2012.

[9] LOZHKIN, A., DYUKINA, N.: *Structurization of analytical geometry on the base of symmetries*, LAP, Saarbruken, 2012.

[10] BACHMAN, F.: *Aufbau der geometrie aus dem spiegelungsbagriff*, Springer-Verlag, Berlin, 1959. (Original in German)

[11] STILLWELL, J.: *Mathematics and Its History*, Springer-Verlag, New York, 2010.

[12] ALEKSANDROV, P.: *Lectures on analytical geometry, supplemented by the necessary information from the algebra*, Ripol-classic, Moscow, 2002.

[13] LOZHKIN, A., ABRAMOV, I., BOZEK, P., NIKITIN, YU.: The issue of calculating elliptic trajectories, *Manufacturing technology*, Vol. 14, No. 4, pp. 561-566, 2014.

[14] BOZEK, P., POKORNY, P., SVETIK, J., LOZHKIN, A., ARKHIPOV, I.: The calculations of Jordan curves trajectory of robot movement, *International journal of advanced robotic systems*, Vol. 7, 2016.

[15] BOZEK, P., IVANDIC, Z., LOZHKIN, A., LYALIN, V., TARASOV, V.: Solutions to the characteristic equation for industrial robot's elliptic trajectories, *Tehnički vjesnik – Technical Gazette*, Vol. 23, No. 4, pp. 1017-1023, 2016.

Review process
Single-blind peer review process.