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INFLUENCE OF THE PRICE MOVEMENTS TO THE ACCURACY WITHIN NUMERICAL PRICE FORECASTING

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Abstract: The paper is aimed at commodity price forecasting using a numerical solution of the Cauchy initial problem for the 1st order ordinary differential equation. To acquire significant forecasting improvement, the idea of the modification of the initial condition value was realized. By having analysed the forecasting success of determined numerical models, it was found out that commodity price evolution affected the accuracy of the price forecasting. The absolute percentage prognoses errors were usually lower at a stable price increase and when price fluctuation appeared. Therefore, prognoses calculated without changing initial condition value were satisfying. Within significant changes in the price evolution and at a rapid price increase, the prognoses acquired higher absolute percentage errors. That caused replacing the initial condition value by the nearest stock exchange. Consider this strategy, the following calculated prognoses got closer to the forecast stock exchanges and price forecasting became more advantageous with respect to the price course.

1 Introduction

Mathematical modelling is one part of the commodity price forecasting and contributes to developing the new branches of solvability of these still current problems [1-4]. In mathematical models forecasting the prices on the commodity exchanges, the statistical methods are often used [5-11]. Our prognostic numerical models were based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations [12-15]. The monthly averages of the daily closing aluminium prices "Cash Seller & Settlement price" presented on the London Metal Exchange (LME) were worked on [16]. As can be seen in Figure 1, the price movements of the aluminium prices on LME (in US dollars per tonne) changed dramatically within the observing period [12-15].

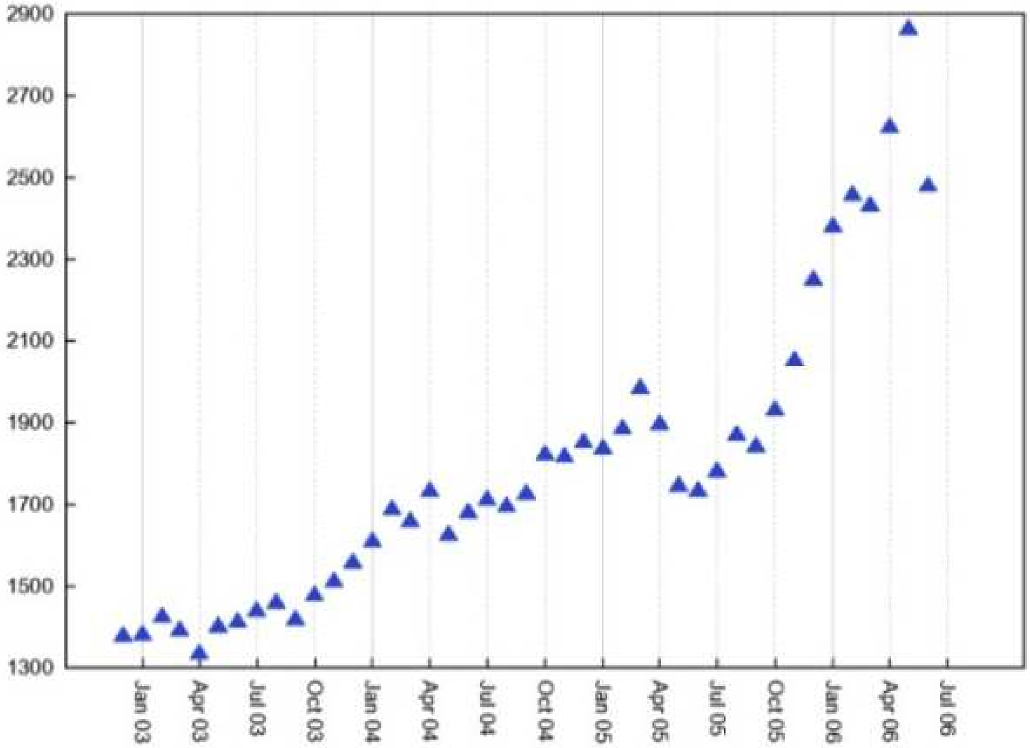

Figure 1 The aluminium stock exchanges on LME in the years 2003 – 2006 [12-15]

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2 Mathematical models

Let us consider the Cauchy initial problem in the form

$$y' = a_1 y, \quad y(x_0) = y_0 \,. \tag{1}$$

The particular solution of the problem (1) is in the form $y = k\, e^{a_1 x}$, where $k = y_0\, e^{-a_1 x_0}$. The price prognoses were created by the following steps (more details in [12-15]):

The 1st step: Approximation of the values – the values of the approximation term were approximated by the least-squares method. The exponential function in the form $\tilde{y} = a_0 e^{a_1 x}$ was used. Let us consider two different variants, variant B and variant E (see Figure 2 and Figure 3).

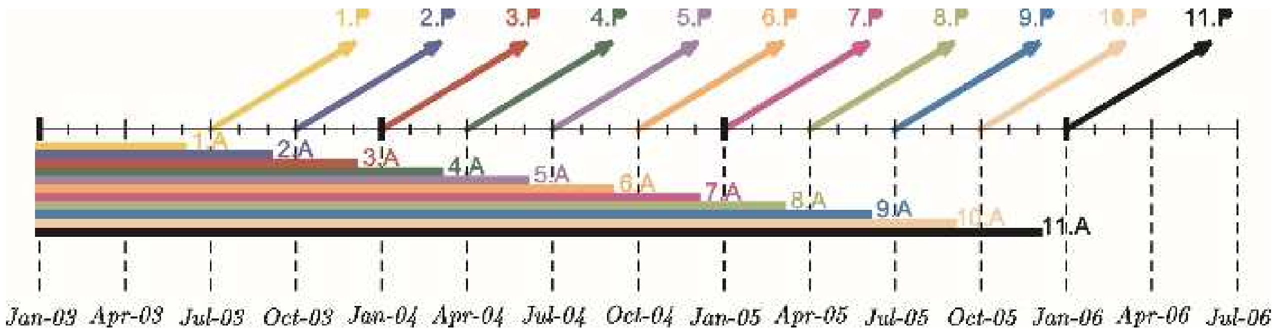

Figure 2 Variant B (A – approximation term,P – forecasting term) [12-15]

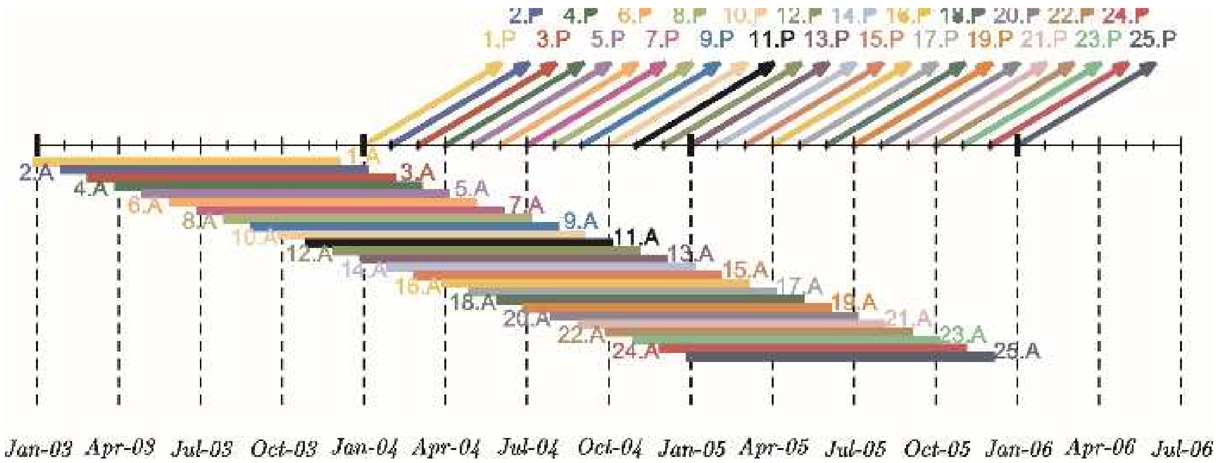

Figure 3 Variant E (A – approximation term,P – forecasting term) [12-15]

The 2nd step: Formulating the Cauchy initial problem – according to the acquired approximation function $\tilde{y}$, the Cauchy initial problem (1) was written in the form

$$y' = a_1 y, \quad y(x_i) = Y_i \,, \tag{2}$$

where $x_i = i$ is the last month of the approximation term, Y_i is the stock exchange in the month x_i.

The 3rd step: Computing the prognoses – the formulated Cauchy initial problem (2) was solved by the numerical method [17]. The method uses the following numerical formulae

$$x_{i+1} = x_i + h \,,$$

$$y_{i+1} = y_i + bh + Qe^{vx_i}\left(e^{vh} - 1\right),$$

for $i = 1, 2, 3, \ldots$, where $h = x_{i+1} - x_i$ is the constant size step. The unknown coefficients are calculated by means of these formulae $v = \dfrac{f''(x_i, y_i)}{f'(x_i, y_i)}$,

$$Q = \frac{f'(x_i, y_i) - f''(x_i, y_i)}{(1-v)\, v^2 e^{vx_i}} \,, \quad b = f(x_i, y_i) - \frac{f'(x_i, y_i)}{v} \,.$$

The prognoses within six month following the end of the approximation term were determined. The daily prognoses in trading days were calculated by chosen numerical method. Their arithmetic mean served for finding the monthly prognosis in each month of the forecasting term (for more details see [12-15]). Let us determine the absolute percentage error [18], $|p_s| = \dfrac{|y_s - Y_s|}{Y_s} . 100\,\%$, where y_s was calculated prognosis, and Y_s was the real stock exchange in the month x_s. The price prognosis y_s is acceptable in practice, if $|p_s| < 10\ \%$. Otherwise, it is named the critical forecasting value. To compare the accuracy of the forecasting of all forecasting terms, the mean absolute percentage error (MAPE) $\bar{p} = \frac{\sum_{s=1}^{t} |p_s|}{t}$ was determined [18], where, in our numerical prognostic models, $t = 6$.

Let us consider three different numerical models. Using the original model, the monthly prognoses in the months x_{i+s}, for $s = 1,2,\dots,6$, were obtained by solving Cauchy initial problem (2) without changing the initial condition value. Although the prognoses obtained by the original model were steeper increasing, the forecasting was not sufficient to accommodate to a steep stable increase or decrease. Also the changes in the price movements caused higher forecasting inaccuracies [15]. Therefore, we were interested in possibilities how to improve the forecasting results by modification of this model. The influence of the initial condition value's change to the forecasting accuracy was observed.

Consider the modified model, the initial condition value in the month x_{i+s}, for $s = 1,2,3,4,5$ was replaced by calculated monthly prognosis y_{i+s}. This strategy gave us similar results as the original model [15]. To improve the forecasting results, the idea of a replacing the initial condition value by aluminium stock exchange was realized. The initial condition value in the month x_{i+s}, $s = 1,2,3,4,5$, was changed either by the calculated monthly prognosis y_{i+s} or by the stock exchange in the month when the absolute percentage prognosis error exceeded the chosen 7 %. Let us named the modification of the initial condition value by the chosen stock exchange as the initial condition drift [12-15]. In the following figures, critical values are red. If the prognosis was not critical value, but its absolute percentage error was greater than 7 %, then that is blue. The prognoses with the absolute percentage error less than 7 % are black.

3 Result and discussion

3.1 The success of numerical models at commodity price forecasting

Within the considered 36 forecasting terms of variants B and E, the success of the determined models was studied. For each forecasting term, the most successful numerical model was defined [15]. The forecasting success of determined mathematical models within forecasting terms is visible in Table 1.

Table 1 The success rate of determined mathematical models

	The original model	The modified model	The model using initial condition drift
Variant B	5	1	6
Variant E	8	4	18
Total	13	5	24

The results show the forecasting by the model using initial condition drift as the most accurate, especially in variant E. This model acquired the lowest MAPE in 19 forecasting terms. In four of them, we obtained the same results by the model using initial condition drift and the modified model. The original model was the most suitable for 12 forecasting terms. In one forecasting term, forecasting results were the same for all chosen models.

Comparing the values of prognoses obtained by the original and the modified models, we have found out that the prognoses determined by the original model were faster changing than prognoses calculated in the modified model. Therefore prognoses of the original model were usually more accurate than prognoses of the modified model [15]. The differences between these two numerical models were too small. Within problematic forecasting terms, these models were such inaccurate that the initial condition drift occurred. The change of the initial condition value by the nearest stock exchanges got calculated prognoses closer to the real stock exchanges and significantly improved forecasting results [12-15].

Let analyze the forecasting success of determined models within different price evolution. The forecasting terms of variants B and E were divided into groups with the same type of the price course. Within these groups, the success of determined models were studied. For each forecasting term, the model with the lowest MAPE was found. The forecasting success of determined mathematical models is visible in Table 2 and Table 3.

Table 2 Distribution of the number of forecasting terms in groups of price movement - variant B

	The original model	The modified model	The model using initial condition drift
Stable price increase	2	0	1
Significant fluctuation	3	1	1
Price decline following the price increase	0	0	2
Price increase following the price decline	0	0	2

Table 3 Distribution of the number of forecasting terms in groups of price movement - variant E

	The original model	The modified model	The model using initial condition drift
Stable price increase	3	0	0
Significant fluctuation	4	3	5
Price decline following the price increase	1	1	6
Price increase following the price decline	0	0	7

The tables show that the model using initial condition drift was the most advantageous in both variants B and E when changes in the price course appeared. Within a price fluctuation, the initial condition drift was not always suitable. The most successful was model with the closest prognoses to the changing values of the stock exchanges. Steeply increasing prognoses of the original model were the most accurate within a stable price increase.

3.2 *The forecasting success of the most accurate model*

The model using initial condition drift was usually the most successful especially within significant changes in price evolution and at a steep price increase.

- **steep price decline following the price increase**

Within these periods, the price decrease was significant. The approximation terms with a price increase belonged to the observed forecasting terms. Therefore, the approximation functions and calculated prognoses were increasing too. Thus, forecasting by both the original and the modified models was not able to accommodate to a steep decline of the stock exchanges. The forecasting without changing the initial condition value by the stock exchange failed. The absolute percentage prognoses errors were higher and caused the initial condition drift [12,13].

Within the period *April 2005 – September 2005* (see [12] and Figure 4) the forecasting without using the initial condition drift failed. The prognosis in month with the highest decline (May 2005) was the most inaccurate. Because the absolute percentage prognosis error exceeded 7 %, the initial condition drift occurred. Using drift, the price decline was captured and a significant improvement in the forecasting was obtained. MAPE of the forecasting term obtained by the model using initial condition drift decreased from 12,55 % (the modified model) to 4,96 % (variant B) and from 12,63 % (the modified model) to 4,94 % (variant E). MAPE for the original model was 14,45 % (variant B) and 14,6 % (variant E) due to higher increased prognoses which were inappropriate when stock exchanges decreased.

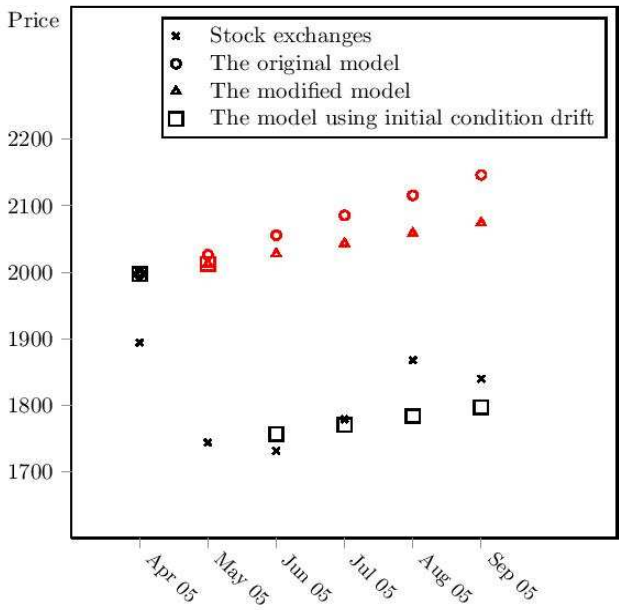

Figure 4 The forecasting by numerical models within April 2005 – September 2005 (variant E)

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- **steep price increase following the price decline**

Due to decreasing stock exhanges in the corresponding approximation terms, the approximation functions had either slowly increasing course or they could be even decreasing. Thus, the calculated prognoses at a steep increase were highly inaccurate. The initial condition drift was necessary for putting the prognoses nearer to a steep price increase [12,13].

Within the period *September 2005 – February 2006* (see Figure 5), the initial condition drift was occurred in the month with a steeper increase of the stock exchange (November 2005). By changing the initial condition value by the nearest stock exchange, the forecasting was able to accommodate to a steep price increase. The forecasting improvement caused decreasing MAPE of observed forecasting term from 12,55 % (the modified model) to 6 % (the model using initial condition drift) [10]. Using the original model, due to higher increase of prognoses, we obtained slightly better forecasting results than by the modified model (MAPE 12,48 %).

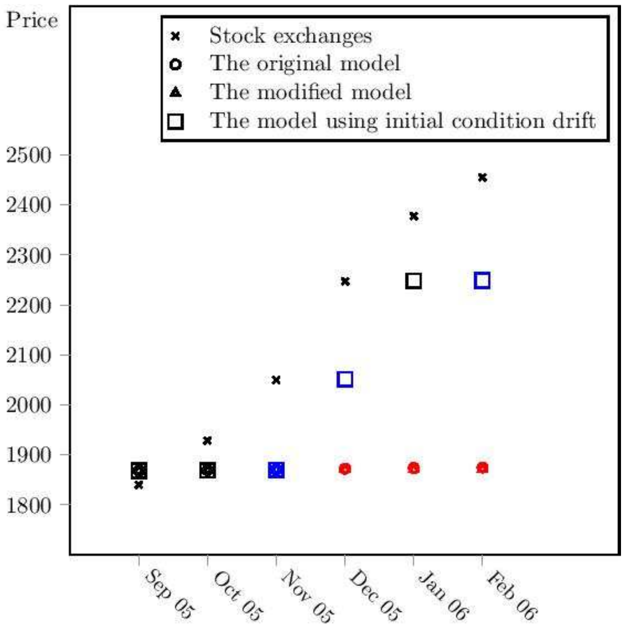

Figure 5 The forecasting by numerical models within September 2005 – February 2006 (variant E)

- **steep stable price increase**

If the price increase was steep, the increase of the forecast stock exchanges was higher than the increase of the stock exchanges within the approximation term. Therefore, the stock exchanges increased faster than calculated prognoses. Thus, the absolute percentage prognoses errors increased with time. If they exceeded 7 %, the ininial condition drift moved the next calculated prognoses closer to steeply increasing stock exchanges. That is a reason why the model using initial condition drift acquired the most accurate prognoses.

Within the period *October 2003 – March 2004* (see Figure 6), the forecasting failed in the third month of the period (December 2003). Using the initial condition drift, all critical forecasting values in the observed period were eliminated. The MAPE decreased from 9,44 % (the modified model) to 4,98 % (the model using initial condition drift) [12]. Larger values of the prognoses acquired by the original model improved forecasting results in comparison with the modified model. But, the increase of prognoses was not sufficient to catch steeply increasing stock exchanges. Thus MAPE of this forecasting term using the original model was 8,92 %.

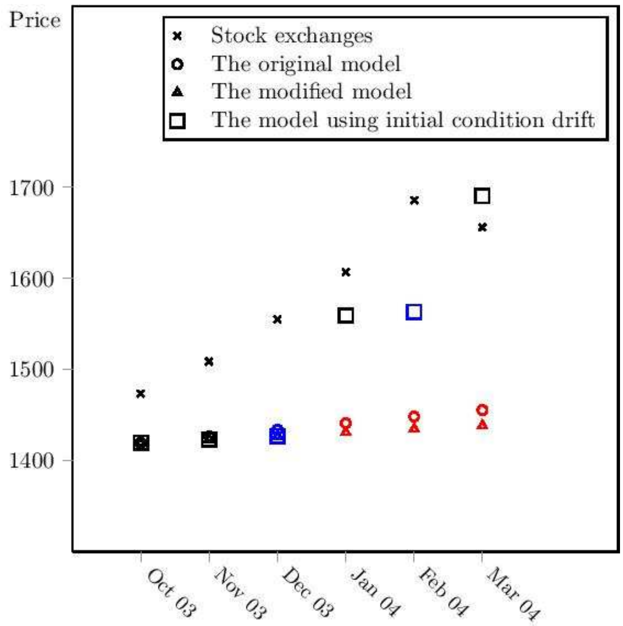

Figure 6 The forecasting by numerical models within October 2003 – March 2004 (variant B)

- **moderate fluctuation**

If moderate fluctuation occurred in forecasting terms, usually the absolute percentage prognoses errors were less than 7 %, so the initial condition value was not replaced by the stock exchange. Thus, forecasting results of both the modified model and the model using initial condition drift were the same [12,13]. In the case, the increased rate of forecast stock exchanges was slower than the increased rate of stock exchanges in approximation term, more suitable was a moderate increase of the prognoses acquired by the modified model than a higher increase of the prognoses obtained by the original model. This situation can be seen in period *April 2004 – September 2004* (see Figure 7), when the most successful were both the model using initial condition drift and the modified model (MAPE: variant B: 1,70 %, variant E: 2,51 %). MAPEs for the original model were 3,34 % in variant B and 4,90 % in variant E.

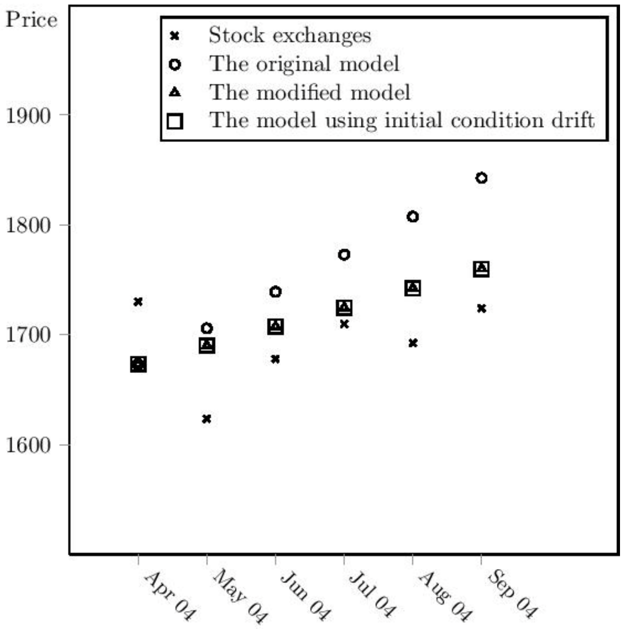

Figure 7 The forecasting by numerical models within April 2004 – September 2004 (variant E)

Nevertheless, the research results indicate low understanding of smart packaging, it is interesting to monitor the innovation status development according to the annual status change for the years 2017 and 2018 (Figure 2), indicating a significant change in the perception of this issue.

Continuously for the youngest monitored age category (15-26 years), smart and active packaging is still attractive. While retaining the innovation status at the same level, the impact of smart and active packaging has changed. It signifies the acceptance of this innovation by a still larger percentage of consumers in this age category. Constantly for this age group, this kind of innovative packaging is still attractive with clear positive impact on customers' satisfaction. The most important and interesting shift of innovation status is in the age category 27-40 years, representing a significant change from the previously negative to the positive innovation status. It means wider acceptance of that kind of innovation and positive improvement of consumers´ attitudes.

Nevertheless, the innovation status of smart packaging in the age category 41 years and older is still negative, a significant positive shift is seen mainly according to category 41-60 years, which means that a higher percentage of customers in this age category perceives them more positively. These changes indicate gradual learning and awareness about active and smart packaging issue also by elder people.

3.3 The forecasting success of the original model

Faster changing prognoses favoured the original model, especially in either a moderate stable or fluctuating price increase.

- **stable price increase**

At a stable moderate price increase, usually the absolute percentage prognoses error was less than 7 %, so the initial condition value was not replaced by the stock exchange. Thus, the forecasting results of the model using initial condition drift were the same as the results of the modified model [12,13]. Because of higher increase of the prognoses of the original model than the prognoses of the modified model, the forecasting by the original model was the most accurate.

This situation could be visible in the period *October 2004 – March 2005* (see Figure 8). The absolute percentage prognoses errors of the modified model were less than 7 %, so the strategy of the modification of the initial condition value by the stock exchange could not improve the forecasting results. MAPE of the forecasting term for the modified model and the model using initial condition drift was the same, 5,14 % in variant B and 5,30 % in variant E. The prognoses obtained by the original model increased faster, so for this model the lowest MAPE 3,63 % in variant B and 3,89 % in variant E was gained.

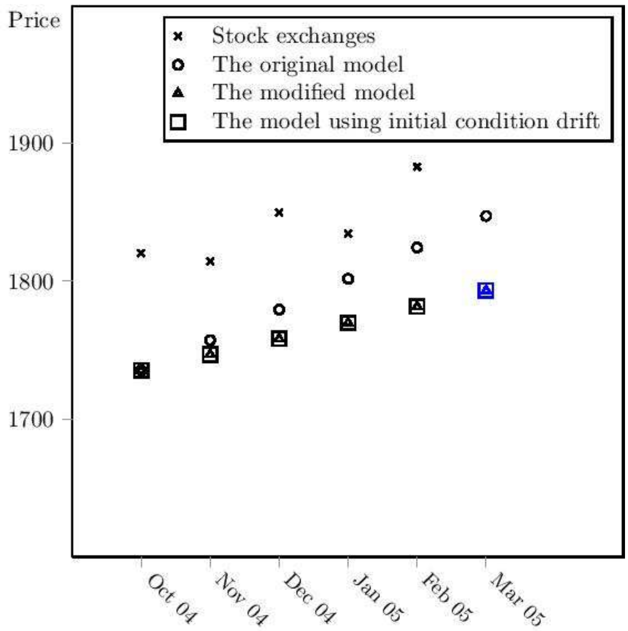

Figure 8 The forecasting by numerical models within October 2004 – March 2005 (variant E)

- **significant price fluctuation**

Within significant price fluctuation the most suitable were models that allowed placing the initial condition value the nearest to the following price evolution. By the model using initial condition drift, the initial condition value was often replaced by the local maximal or minimal value. That was usually not suitable for the forecasting of the following unstable price movements [12].

Within the period *January 2004 – June 2004* (see Figure 9) fluctuating increase was occurred. The forecast stock exchanges increased faster than the stock exchanges in the approximation term. Therefore the calculated prognoses were not enough accurate and the initial condition drift was realized. The initial condition value was changed by the stock exchange in the month with local maximal value (the fourth month, April 2004). Since the price increase did not continue, the calculated prognoses using the initial condition drift were the most inaccurate. MAPE of the forecasting term for this model was 5,40 % [12]. Better forecasting results were obtained without changing the initial condition value by the stock exchange. MAPE of the forecasting term was 3,72 % for the original model and 4,81 % for the modified model.

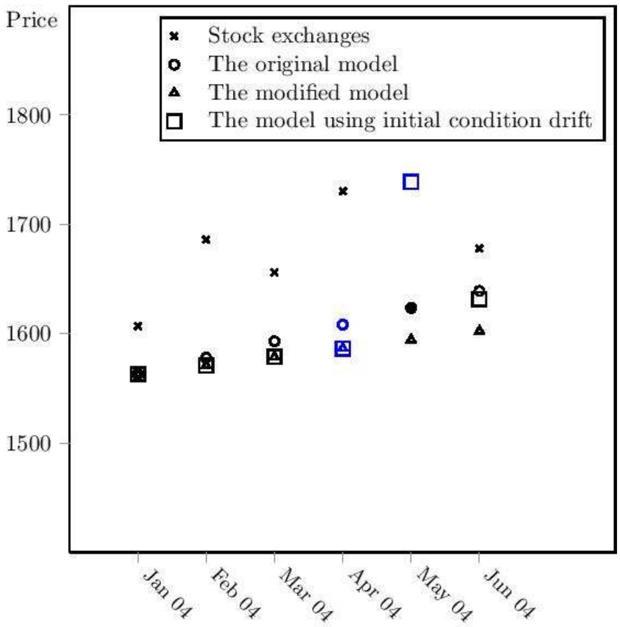

Figure 9 The forecasting by numerical models within January 2004 – June 2004 (variant E)

Within a moderate fluctuation increase in the period *June 2004 – November 2004* (Figure 10), the forecasting was so accurate that the absolute percentage prognoses errors were small and the initial condition value was not modified by the stock exchange. Therefore, higher increasing prognoses of the original model were more accurate than the same prognoses of both the modified model and the model using initial condition drift, which MAPE was 3,40 %. The most accurate was the original model with MAPE 1,59 %.

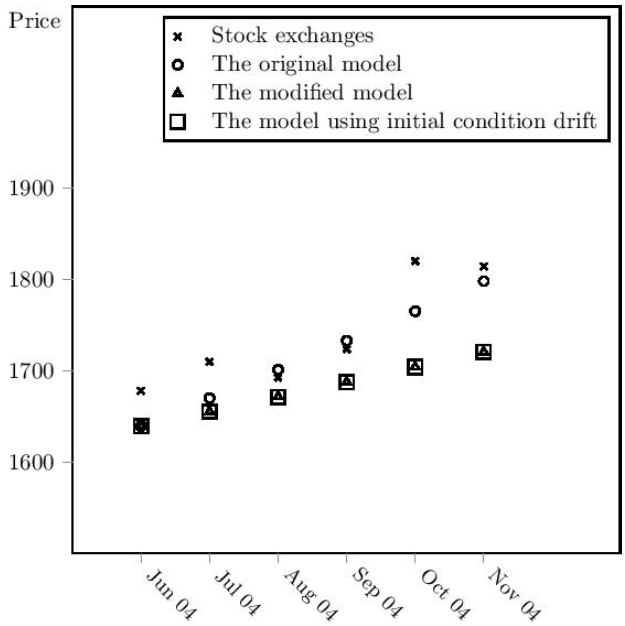

Figure 10 The forecasting by numerical models within June 2004 – November 2004 (variant E)

4 Conclusion

By having analyzed the forecasting reasults of determined numerical models, it was found out that price evolution influenced the forecasting accuracy of the models. Within the most problematic forecasting periods, the model using initial condition drift was usually the most advantageous. We recommend to use this model, especially during significant changes in the price evolution and at a steeper price increase. The idea of the modification of the initial condition value by the stock exchange during the numerical solution of the determined Cauchy initial problem, significantly improved the forecasting accuracy. That allowed to put calculated prognoses closer to the real price movements and made the forecasting more accurate than the forecasting by either the original model or the modified model. Within price fluctuations, due to replacing the initial condition value either by the local maximal or local minimal stock exchange, the initial condition drift was not always advantageous. Also, when the initial condition drift did not occur, higher increased prognoses of the original model were more advantageous than the prognoses determined by the modified model. Using the most appropriate type of the mathematical models within different price evolution made price forecasting more accurate.

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