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FACILITY LOCATION MODEL WITH INVENTORY TRANSPORTATION AND MANAGEMENT COSTS

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Abstract: This work is focused on the integration of the standard EOQ (Economic Order Quantity) model within the facility location decision model. This is proposed to extend on the facility location task which is usually performed based on just the overall demand of the customer locations to be served. If the inventory costs are considered within the demand supply process, these may affect the overall transportation costs as these are not linearly dependent of the demand. As such, the extended model considers, besides the distances, performance and capacity of the vehicles, the order quantities and the period in which they should be fulfilled. This model was tested with a reference instance of 200 suppliers and one distribution centre. The distances were estimated by considering the geographical locations of all elements in the network and the spherical model of the Earth's surface to obtain the metric in kilometres. As analysed, by considering the inventory costs within the facility location model, it can lead to refine the location to obtain long-term savings in transportation.

1 Introduction

Several efforts have been performed to improve supply chain operations, developing practical tools and instruments that have contributed to company competitiveness and also for researchers who seek to provide real and useful results. This has involved the use of mathematical decision models for the optimization of resources. Among these models, some have contributed to the design of distribution networks by integrating the modelling of real factors such as demography data, times, capacities, speeds, and restrictions. The importance of solving problems related to distribution networks lies in the fact that all services and products require them for an efficient delivery to customers and industries. Thus, mechanisms to reduce delivery and production times, improve quality and reduce waste, are frequently sought.

Recently, research has been focused on the integration of location and inventory decisions in a single framework, with the motivation that integration can bring substantial cost savings. Integrated supply chain network design involves several core components among which facility location and inventory management are the main components [1]. The benefit of the integration depends on the relative size of the facility location costs and inventory management costs. Since traditionally facility location decisions are made before the inventory policy is decided, the benefit of integration increases as inventory costs increase [2].

In this work we describe an integrated inventory-facility location model to minimize distance, transportation and inventory management costs. For this, the proposed model considers vehicle-dependent fuel consumption costs and management-dependent inventory costs to adjust the demand of each customer to be gathered by a distribution centre.

The paper is structured as follows: in Section 2 the technical background and research from the literature is presented. Then, the justification of the research is discussed in Section 3. In Sections 4 and 5 the development of the integrated distribution-inventory model and the results on the test instance are presented and discussed. Finally, the conclusions are presented in Section 6.

Nowadays it is very important that a plant or facility, independently of its qualification or size, has a strategic location according to its supplies and customers. In general, its resources must be as close as possible for the purpose of avoiding setbacks that generate high repair costs, so that the raw material arrives on time. In general, the location of a plant is strategic for its success, and the employees must have the appropriate infrastructure and means of transportation.

No less important is the subject of inventories. It is almost as important as the investment decision itself, since it is fundamental for the development of a company. It is a strong economic investment that will remain on hold or with little movement. For this reason, good management and planning of inventories is of vital importance to avoid financial problems in the future. Hence, it is the area where bad planning can impact the most.

This is why in this work we considered the unification of these contexts. If each one separately contributes to the improvement and optimization of resources, a unified model can lead to more significant improvements. Thus, the intention is to verify that the financial impact, the optimization of resources (in this case fuel) and the distance between the distribution centre and the customers, are key factors for the adequate economic management.

The main objective of this work is to find the strategic location of a new plant with the intention of minimizing the distances respect to its suppliers. This can lead to savings in transportation costs by decreasing fuel consumption for each location, properly planning inventory levels and avoiding additional costs by determining the exact amounts in the established times to prevent unnecessary transfers.

2 State of the art

2.1 *Facility location problem*

Alfred Weber, a German economist, developed one of the earliest approaches to determine the location of manufacturing industries [3]. The facility location problem (FLP) was introduced by him in the early 20th century. This model is focused on determining the location of a warehouse in the two-dimensional plane with customers located across the plane [4]. This location must minimize the total distance from the warehouse to n customers located at different points. Weber assumed that the warehouse could be located anywhere in the plane. Hence, this objective function can be formulated as [4]:

$$\boldsymbol{min} \sum_{i=1}^{n} \boldsymbol{d}(\boldsymbol{x}, \boldsymbol{x_i}) \boldsymbol{w_i}, \boldsymbol{s.t.} \, \boldsymbol{x} \in \mathbb{R}^2 \tag{1}$$

Where $x \in \mathbb{R}^2$ is the warehouse or plant location, $x_i \in \mathbb{R}^2$ are the locations of the n customer points (hence, $i = 1, \ldots, n$), and w_i a weight associated to the customer point i (commonly, this weight is used to provide a priority for a given point).

This problem has received much attention in the literature and industry. Researches have been performed to complement this model with more variables. Weber considers a uniform plain where it is always possible to go directly from one point to any other at a given rate. In this two-dimensional space, the market site, the location of "localised" resources, and the spatial distribution of labor are regarded as given [5]. The Theory of Location has adopted a unilateral approach to define the locational optimum. While the "minimum cost" concentrates on the spatial variation of costs by paying little attention to demand as a primary locational force, the approximation of the market area overes-timates demand at the expense of cost variations [6].

Despite the long history of the single-facility location problem, there still appears to be much to say about it as evidenced by several recently published articles on this topic [7]. For example, Jamalian & Salahi [8] considered the multi-facility Weber location problem (MFWP) with uncertain location of demand points and transportation cost parameters. Uno et al. [9] considered the Weber location problem with weights including both uncertainty and vagueness. By representing its weights as fuzzy random variables, it was extended to a fuzzy random weighted Weber problem, and then formulated as a fuzzy random programming problem. On the other hand, Murat et al. [10] developed an efficient exact "shooting" algorithm with capacity acquisition problem and dense demand.

As presented, this is a very interesting and extensive research topic, and many researchers have tried to find the solution for so many associated variables supported by this model. Hence, it is important to note that an "optimal" location for any plant, or house, or city, or company, always is influenced by many factors, and some of them are explored in this work.

2.2 *Economic Order Quantity (EOQ)*

When Ford W. Harris published his short three-page article in the Economic Order Quantity model in 1913, he likely did not foresee that it would still be discussed and used 100 years later [11]. Technically, his model determines manufacturing quantities that minimize inventory-associated costs [12].

The EOQ (Economic Order Quantity) proposes an inventory planning based on policies that will achieve an optimal investment. Many inventory planning models are available and basically all of them try to answer the following two questions [13]:

- How much to order?
- When to order?

The corresponding model of Harris is formulated as:

$$\boldsymbol{Q} = \sqrt{\frac{\boldsymbol{2DCo}}{\boldsymbol{Ch}}} \tag{2}$$

Where Co is the standard ordering cost, D is the total demand per unit of time (through-out a planning horizon), Ch is the holding cost per unit of product, and Q the periodic lot size that minimizes the inventory costs within the planning horizon [14]. The fol-lowing formulation estimates the total inventory costs associated to a lot quantity Q where each unit has a cost C:

$$\boldsymbol{TC} = \left(\frac{\boldsymbol{D}}{\boldsymbol{Q}}\right) \boldsymbol{Co} + \left(\frac{\boldsymbol{Q}}{\boldsymbol{2}}\right) \boldsymbol{Ch} + \boldsymbol{DC} \tag{3}$$

Eq. (2) and Eq. (3) have been extended in different forms to model the diverse contexts of the inventory supply strategies. Rossi et al. [15] proposed an approach that regulates the processing of different items by a shared resource according to a control model based on an ordering policy that combines the EOQ with a policy based on minimum and maximum inventory levels (min–max policy). Another interesting study about the EOQ model is the one reported by Dobson et al. [16] that proposed the inventory management decision model for a retailer selling a single perishable good in a deterministic setting. This model considered into account the consumers' assessment of quality over the lifetime of the products and assumed that the demand rate was a linearly decreasing function of the age of the products.

As presented, this model can have many scenarios and variables, and these will continue to grow as the industrial and inventory policies become more complex.

2.3 *Arch length on the sphere*

In order to find the distance between the plant or distribution centre *(i)* and its customers or suppliers *(j)*, we used the following mathematical formulation:

$$\boldsymbol{d_{ij} = R\, Arcos(sin\varphi_i sin\varphi_j + cos\varphi_i cos\varphi_j cos(\lambda_i + \lambda_j))} \quad (4)$$

Where (φ_i, λ_i) and (φ_j, λ_j) are the geographical latitude and longitude in radians of two points on the sphere, and R is the sphere radius (see Figure 1). If distances on the spherical Earth are to be estimated, then R = 6371 km.

The spherical model allowed the estimation of a more suitable distance metric in contrast to the Euclidean distance. This is because the spherical model is more similar to the Earth's true form than the flat form assumed by the Euclidean model. Nevertheless, it is important to mention that the truer form of the Earth's surface is defined as a geoid, which can be better modelled by an ellipsoid.

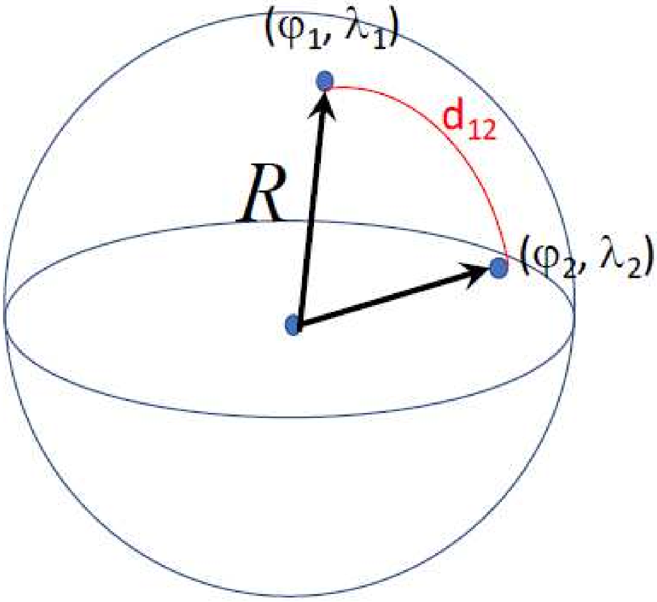

Figure 1 Spherical model of the Earth: Arc length between two points on the surface

2.4 *Non-linear programming*

In mathematics and software engineering, non-linear programming is defined as the process of solving an optimization problem where the objective function to be maximised or minimised, and / or some of its constraints (restrictions) are non-linear. Currently, many software tools are available for optimization of problems with this characteristic (i.e., Microsoft's Solver, and LINDO). Due to the nature of Eq. (4), the proposed facility location model is a non-linear problem.

3 Development of the model and test instance

To begin with, the development of the model, first we begin by knowing its assumptions:

- Location of a single new plant.
- 200 suppliers are considered for the new plant with locations around the Mexican Republic.
- The Facility Location Problem was considered with discrete data (i.e., each data is different and independent from the other).
- Initially, only distance and demand are considered with no other urban and infrastructure factors.
- Based on the previous assumption, the global coordinates of the main locations are determined.

- Each supplier provides a unique product with different characteristics and prices than the other suppliers.
- 6 different types of vehicles are handled for the 200 suppliers, with different fuel efficiency per kilometre.
- Each supplier has one of the six types of vehicles previously considered.
- The weight of the lot never exceeds the vehicle capacity.

After knowing all of these assumptions, the second step was to determine the geographical locations of the 200 points around the Mexican Republic. The geographical coordinates of these locations (in degrees) were converted into decimal coordinates for the estimation of Eq. (4). Then, the third step consisted on determining the demand for each location (except for the distribution centre), including quantity of pieces, weight in kilograms (by unit product) and unit prices. This is a regular and stable demand forecasted for a planning horizon of 10 years with an annual increment of 5.0%.

In the fourth step we determined the characteristics of each vehicle. Table 1 presents this information.

Table 1 Characteristics and performance of each vehicle (own work from data available in [17])

Vehicle Type *k*	Maximum Capacity (kg)	Diesel Efficiency (km/litre)
V1	17000	9
V2	28000	6
V3	31000	6
V4	27000	4.5
V5	25000	2.5
V6	48000	2.5

To have a complete calculation, the actual fuel price was included to obtain the efficiency for each travelled kilometre. From [18] the cost of $18.87 per litre was determined. Once the diesel cost per litre is obtained, a cost per travelled kilometre CV_k is estimated for each vehicle *k* as follows:

$$CV_k = 18.87\ \$/lt \times (1/Diesel\ Efficiency\ in\ km/lt\ of\ vehicle\ V_k) \qquad (5)$$

In example, for vehicle V_3, the cost per travelled kilometre is $CV_3 = 18.87 \times (1/6) = \$3.145/km$. With this metric for CV_k, the fifth step consist on determining the distance in kilometres between the distribution centre and each customer or supplier location by means of Eq. (4). Note that by multiplying Eq. (4) by Eq. (5) a "distance cost" between two locations can be estimated (if the vehicle returns to the supplier after delivery at the distribution centre, then it is assumed that the distance cost is performed twice).

How many times the distance cost will take place depends of the quantity of inventory lots to be delivered at the distribution centre through a planning horizon. This is estimated by means of the EOQ policy as:

$$n = \frac{D}{Q} \qquad (6)$$

With all of these calculations, we can define an "inventory transportation cost", which is the very important metric to complement the objective function of the Weber model. The integrated objective function of the extended Weber model then can be expressed as:

$$min \sum_{i=1}^{n} 2d(x, x_i) CV_K n Q_i \qquad (7)$$

In the next section, the implication of this updated objective function is analysed and discussed.

4 Analisys of results

Figure 2 presents the location of the suppliers and the distribution centre considering just the basic model described by Eq. (1). The location was determined with the Solver tool. Observe how the distribution of locations fit the geographical shape of the Mexican Republic. This visualization was obtained by a code developed with the programming platform Octave.

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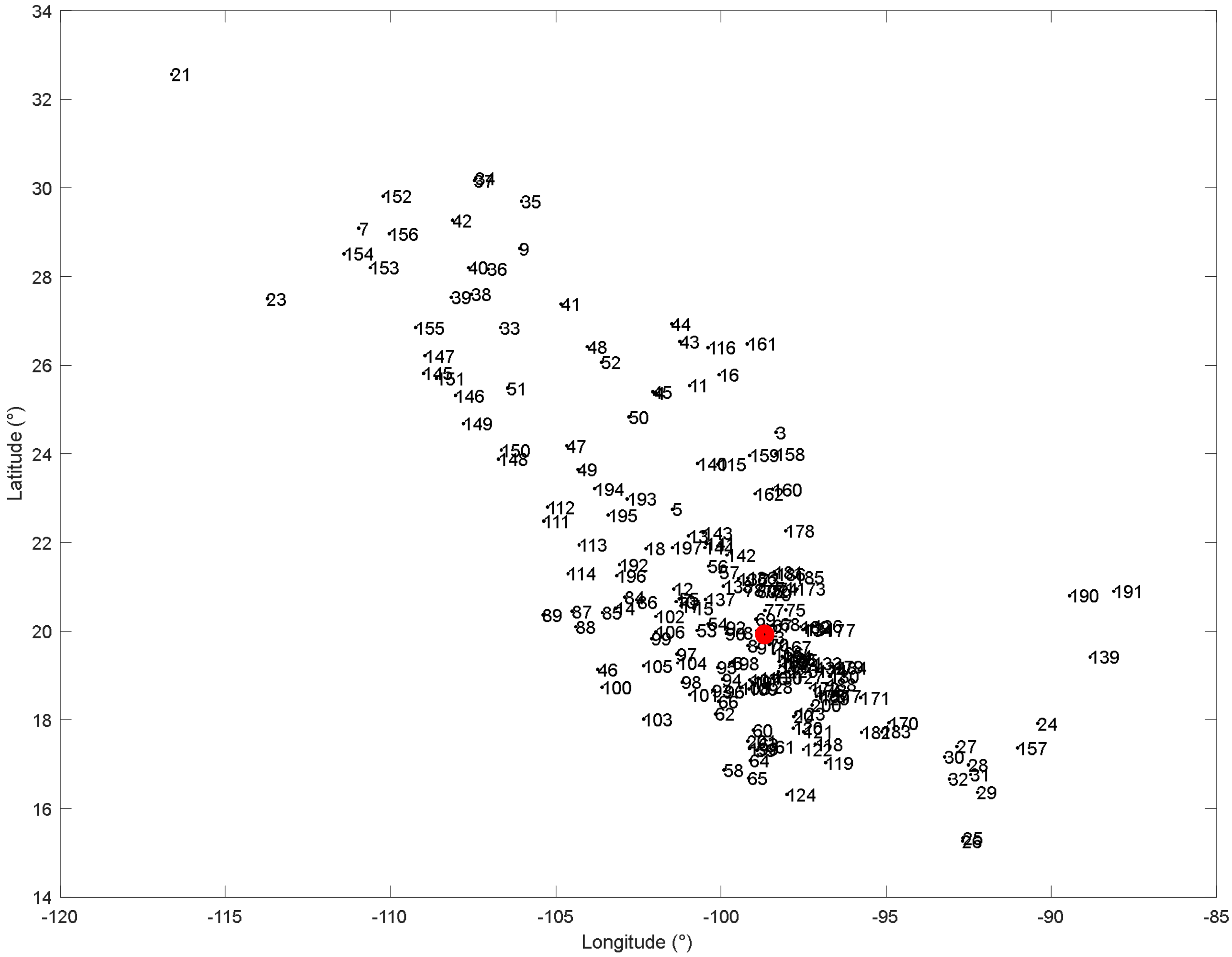

Figure 2 Locations of the suppliers and the plant or distribution centre

Minimization of the basic objective function led to determine the geographical coordinate -99º, 20º as the optimal location for the plant obtained by Eq. (1). Specifically, the location is near the city of Mexico as presented in Figure 3.

Then, we proceeded to re-estimate the location of the plant with the extended model described by Eq. (7). This led to a revised location which was determined at the coordinate -98.68º, 19.91º. Table 2 presents the comparison of economic results for supplier #30 with the standard and revised locations while Figure 4 presents the distance between both plants.

As presented, in both cases distance is minimised, however the economic implications are different. The extended model led to a 10-year transportation cost associated to the revised location for the plant of $1,283,199.00. In contrast to the location estimated with the standard facility location model, this represents a saving of $1,340,911.93 - $1,283,199.00 = $57,712.93.

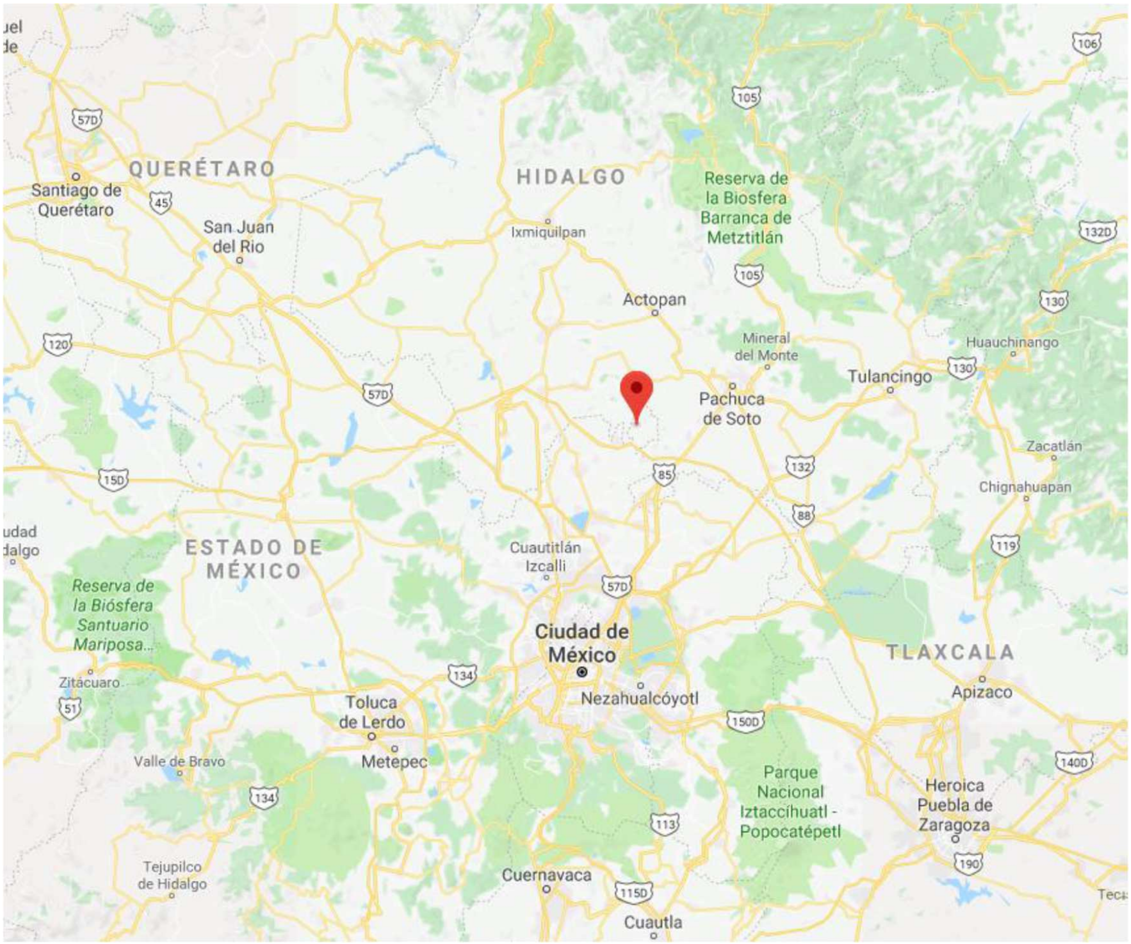

Figure 3 Location of the plant as determined by Eq. (1)

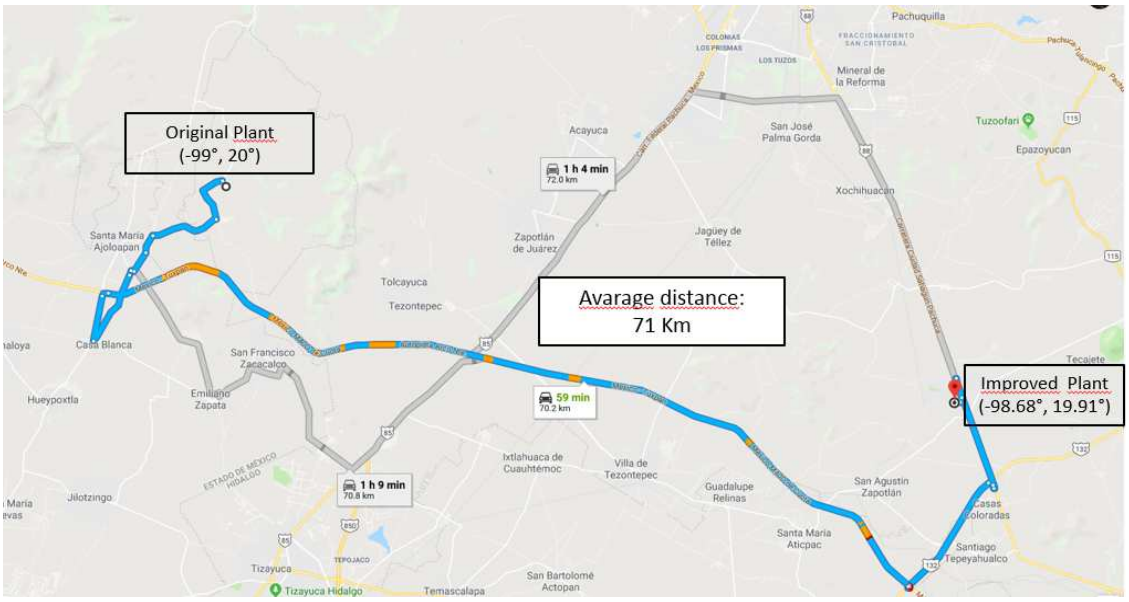

Figure 4 Functional location plants determined by Eq. (1) and Eq. (7)

Table 2. Comparison of results: standard facility location vs. extended model for supplier #30 (own work)

Concept	**Coordinates (Eq. (1)) -99º, 20º**	**Revised Coordinates (Eq. (7)) -98.68º, 19.92º**
Annual demand (USD)	$93,816.80	
Unit cost (USD)	$553.00	
Annual demand (Units) = Annual demand (USD)/ Unit cost (USD)	$170	
Total demand (10 years) with 5% annual increase	2,241	
Unit weight (kg)	4	
Vehicle	V_1	
Vehicle's capacity (kg)	25,000	
Vehicle's performance (km / lt)	2.5	
Diesel cost per litre (USD)	$0.96	
Cost per kilometre	$0.38	
Holding cost (Ch)	$82.95	
Order cost (Co)	$991.00	
Lot size (Q_{30})	231	
Total weight of the lot Q_{30} (kg)	925.51	
Cost of lot Q (USD)	$127,951.41	
Number of orders with the 10 year-period (n=D/Q)	10	
Distance to plant (km)	**779.267**	**745.727**
Distance cost to plant (USD)	**$299.24**	**$286.36**
Transportation cost (Eq. (7) with *i*=30 and k=1)	**$1,340,911.93**	**$1,283,199.00**

An important issue regarding these costs is that the future cost variation and increase of the gasoline are not considered. Hence, savings can be more significant when considering these factors.

5 Conclusion and future works

How important would be the management of inventory if the company belongs to the pharmaceutical industry? If the plant produces medicines with different elements from a set of suppliers, effective delivery of these elements is crucial for the production process. Also, costs associated to delivery of raw material, distribution of the final products, and inventory costs, are crucial to keep competitiveness and good quality products. As discussed in this work, having an appropriate integrated model can lead to significant economic savings. These can be used to reinvest strategically in some other aspects of the supply chain.

Of course, within these costs there are many more factors. However, the proposed model can be the guideline to extend on a more comprehensive approach to optimize transportation within the economic aspects of inventory management. As an example, strategies for safety stock, variable delivery time and dynamic reorder point, can be considered to improve the planning of materials.

Similarly, in terms of transport, the structure of the roads, the size of the vehicles, the size of the orders, the dimensions, the payment of tolls, the maintenance of equipment, etc., influence the determination of operative costs. Thus, this work can be extended to include many more variables to optimize more resources of the supply chain. The following extensions are considered for future work:

- Consider variable demand and uncertainty within the delivery time.
- Consider a different inventory model with uncertain demand (i.e., periodic or continuous review model) as the base inventory strategy.
- Integrate variable costs within the planning horizon as a decision variable.

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