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THE ANALYSIS OF THE COMMODITY PRICE FORECASTING SUCCESS CONSIDERING DIFFERENT LENGTHS OF THE INITIAL CONDITION DRIFT

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Abstract: In the paper the numerical model based on the exponential approximation of commodity stock exchanges was derived. The price prognoses of aluminium on the London Metal Exchange were determined as numerical solution of the Cauchy initial problem for the 1st order ordinary differential equation. To make the numerical model more accurate the idea of the modification of the initial condition value by the stock exchange was realized. By having analyzed the forecasting success of the chosen initial condition drift types, the initial condition drift providing the most accurate prognoses for the commodity price movements was determined. The suggested modification of the original model made the commodity price prognoses more accurate.

1 Introduction

Observing trends and forecasting movements of metal prices is still a current problem. There are a lot of approaches to forecasting price movements. Some of them are based on mathematical models. Forecasting prices on commodity exchanges often uses the statistical methods that need to process a large number of historical market data [6]. The quantity of needed market data can sometimes be a disadvantage. In such cases, other mathematical methods are required.

In our prognostic model numerical methods were used. Their advantage is that, in comparison with statistical models, many fewer market data are needed. Our numerical model for forecasting prices is based on the numerical solution of the Cauchy initial problem for the 1st order ordinary differential equations [1-4].

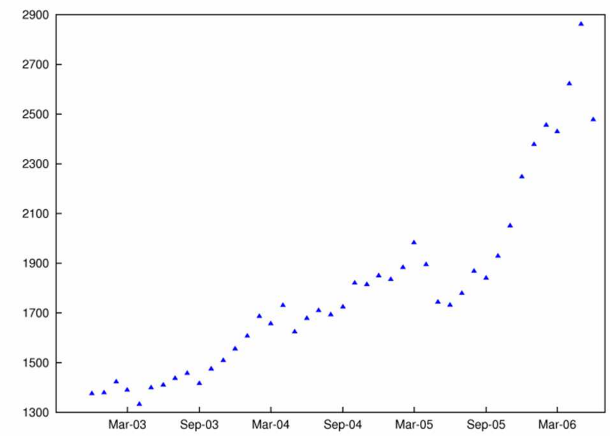

Figure 1 Course of aluminium prices on LME in the years 2003 – 2006

The aluminium prices presented on the London Metal Exchange (LME) were worked on. We dealt with the monthly averages of the daily closing aluminium prices "Cash Seller&Settlement price" in the period from December 2002 to June 2006. The market data were obtained from the official web page of the London Metal Exchange [7]. The course of the aluminium prices on LME (in US dollars per tonne) within the observing period is presented in Figure 1.

2 Mathematical model

We considered the Cauchy initial problem in the form

$$y' = a_1 y, \;\; y(x_0) = y_0 \qquad (1)$$

The particular solution of the problem (1) is in the form $y = k\, e^{a_1 x}$, where $k = y_0 e^{-a_1 x_0}$. The considered exponential trend was chosen according to the test criterion of the time series' trend suitability. The values $\ln(Y_{i+1}) - \ln(Y_i)$, for $i = 0,1,\ldots,42$ have approximately constant course, where Y_i is the aluminium price (stock exchange) on LME in the month x_i. The price prognoses were created by the following steps:

The 1st step: Approximation of the values – the values of the approximation term were approximated by the least squares method. The exponential function in the form $\tilde{y} = a_0 e^{a_1 x}$ was used. When observing the influence of the approximation term length on the prognoses accuracy, we found out that the prognoses obtained by longer approximation terms were more accurate [1], [3]. Let us consider two different variants.

Variant B: The values from the period January 2003 – June 2003 were approximated. The next approximation terms were created by sequential extension of this period by 3 months. Thus the duration of the approximation

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terms was extended (the n^{th} approximation term has $6+3(n-1)$ stock exchanges) (Figure 2).

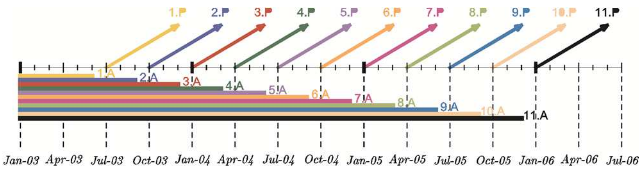

Figure 2 Variant B
(A – approximation term, P – forecasting term)

Variant E: We approximated values within 12 months and each term was shifted by 1 month (Figure 3). (The first approximation term was January 2003 - December 2003.)

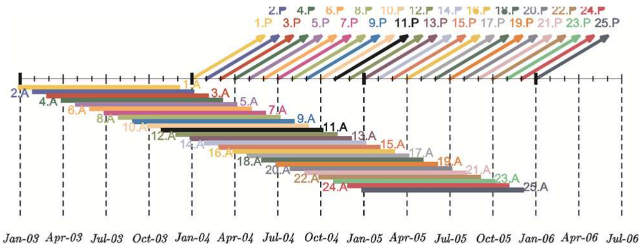

Figure 3 Variant E
(A – approximation term, P – forecasting term)

36 forecasting terms of the original model in both variants B and E within period from July 2003 to June 2006 were observed. From among all forecasting terms, 11 of them belonged to variant B and 25 ones were part of variant E, whereby 9 forecasting terms were common for both variants.

The 2nd step: Formulating the Cauchy initial problem – according to the acquired approximation function $\tilde{y}$, the Cauchy initial problem (1) was written in the form

$$y' = a_1 y, \quad y(x_i) = Y_i \qquad (2)$$

where $x_i = i$ and Y_i is the aluminium price on LME in the month x_i, which is the last month of the approximation term.

The 3rd step: Computing the prognoses – the formulated Cauchy initial problem (2) was solved by the numerical method based on the exponential approximation of the solution. A detailed solution method is seen in [5]. The method uses the following numerical formulae (3.1), (3.2):

$$x_{i+1} = x_i + h \qquad (3.1)$$

$$y_{i+1} = y_i + bh + Qe^{vx_i}\left(e^{vh} - 1\right) \qquad (3.2)$$

for $i = 1, 2, 3, \ldots$, where $h = x_{i+1} - x_i$ is the constant size step.

The unknown coefficients are calculated by means of these formulae (4.1), (4.2):

$$v = \frac{f''(x_i, y_i)}{f'(x_i, y_i)}, \quad Q = \frac{f'(x_i, y_i) - f''(x_i, y_i)}{(1-v)\, v^2 e^{vx_i}} \qquad (4.1)$$

$$b = f(x_i, y_i) - \frac{f'(x_i, y_i)}{v} \qquad (4.2)$$

If we consider the Cauchy initial problem (2), the function $f(x_i, y_i)$ has the form $f(x_i, y_i) = a_1 y_i$ and then $f'(x_i, y_i) = a_1 y'(x_i) = a_1^{\,2} y_i$, $f''(x_i, y_i) = = a_1^2 y'(x_i) = a_1^3 y_i$.

We calculated the prognoses within six months that follow the end of the approximation term in this way:

- **The first month prognosis was** determined by solving the Cauchy initial problem in the form (2). The interval $\langle x_i, x_{i+1} \rangle$ of the length $h = 1$ month was divided into n parts, where n is the number of trading days on LME in the month x_{i+1}. We got the sequence of the division points $x_{i0} = x_i$, $x_{ij} = x_i + \frac{h}{n} j$, for $j = 1, 2, \ldots, n$, where $x_{in} = x_{i+1}$. For each point of the subdivision of the interval, the Cauchy initial problem in the form (2) was solved by the chosen numerical method. In this way we obtained the prognoses of the aluminium prices on single trading days y_{ij}. By calculating the arithmetic mean of the daily prognoses we obtained the monthly prognosis of the aluminium price in the month x_{i+1}.

  So, $y_{i+1} = \dfrac{\sum_{j=1}^{n} y_{ij}}{n}$.
- **The prognoses for the following months** were calculated after modification of the initial condition value. The initial condition value in the month x_{i+s}, $s = 1, 2, 3, 4, 5$ was replaced either by the calculated monthly prognosis y_{i+s} or by some aluminium stock exchange (in case of higher absolute percentage error of given monthly prognosis y_{i+s}). The Cauchy initial problem $y' = a_1 y$, $y(x_{i+s}) = y_{i+s}$, respectively $y' = a_1 y$, $y(x_{i+s}) = Y_p$ (where Y_p is chosen aluminium stock exchange) was used for calculating daily prognoses and their arithmetic mean served to define the monthly price prognosis y_{i+s+1} for the month x_{i+s+1}.

By comparing the calculated prognosis y_s in the month x_s with the real stock exchange Y_s, the absolute percentage error $|p_s| = \dfrac{|y_s - Y_s|}{Y_s} .100\,\%$ is determined. The price prognosis y_s in the month x_s is acceptable in practice, if $|p_s| < 10\,\%$. Otherwise, it is called the critical forecasting value of. To compare the accuracy of forecasting of all forecasting terms, the mean absolute percentage error (MAPE) $\bar{p} = \frac{\sum_{s=1}^{t} |p_s|}{t}$ is determined, where, in our case, $t = 6$.

The modification of the initial condition value by the real aluminium stock exchange price was called the initial condition drift. Let us name the selected minimal absolute percentage error of the prognosis, causing the initial condition drift, the limiting value error. The month in which the absolute percentage error of the prognosis has at least the limiting value error was considered as the limiting month.

The limiting value error of the size 7 % was chosen. Three types of the initial condition drift with regard to their length were considered, namely one-month drift, drift before the limiting month and drift to the limiting month. One-month drift was the shortest chosen initial condition drift, where the initial condition value was replaced by the stock exchange Y_{i+p}, $p = 1, 2, 3, 4, 5$ in the month x_{i+p}, where x_i was the last month of the approximation term and p was the initial condition drift order in the forecasting term. Using drift before the limiting month, the initial condition value was replaced by the stock exchange Y_{L-1} in the month x_{L-1}, where x_L was the limiting month and by means of drift to the limiting month the stock exchange Y_L in the month x_L changed the initial condition value.

3 Results

3.1 The success of the initial condition drifts with different lengths at commodity price forecasting

We started from the original model calculating the prognoses within six months following the approximation term after modification of the initial condition value by the obtained monthly price prognoses [2], [3], [4]. The original model forecasted the aluminium price reliably (the absolute percentage errors were less than 10 %) within the stable price course, when the price did not changed rapidly. Within the rapid increase or decrease of stock exchanges, but also in the case of changes in the price course the forecasting failed. Since the variability with rapid and sudden changes is typical of the commodity price course, we judged the possibility of making the forecasting more accurate by using the modification of the initial condition value by aluminium price.

Within the studied group of 36 forecasting terms, the forecasting within 14 of them was so accurate that the initial condition drift did not occur. The initial condition values were replaced just by calculated monthly prognoses. Since in the remaining 22 forecasting terms the forecasting was less accurate, some of the prognoses gained the absolute percentage error higher than chosen limiting value error, and the initial condition drift occurred. Therefore the forecasting results differ from the original model.

Within each forecasting term for the variants B and E, three different lengths of the initial condition drift were taken into account. For each forecasting term, in which the initial condition drift was occurred, we defined the type of the drift's length to obtain the most accurate forecasting results. (The lowest MAPE of the forecasting term was gained.) The following table shows the number of the forecasting terms in which the forecasting by the determined types of the initial condition drift was the most accurate.

Table 1 The comparison of the success rate of the chosen types of the initial condition drift

Variant	The initial condition drift did not occur	The type of drift		
		One-month drift	Drift before the limiting month	Drift to the limiting month
B	4	1	1	5
E	10	3	1	13
Total	14	4	2	18

With regard to the initial condition drift length, the most accurate forecasting results were obtained in both variants B and E by using drift to the limiting month, in other words, the longest initial condition drift. This type of drift had the lowest MAPE in eighteen forecasting terms. In two of them we obtained the same results by using one-month drift (the initial condition drifts were the same). Shorter initial condition drifts were far less successful. One-month drift was the most advantageous for four forecasting terms, two of them were already mentioned in the previous drift type. Drift before the limiting month was the most accurate only in two forecasting terms. The success rate of the determined types of the initial condition drift was analyzed within different moves of the aluminium price course and was demonstrated at the specific forecasting terms while considering commodity price evolution.

3.2 The forecasting success of the longest initial condition drift

The longest initial condition drift was the most advantageous. It acquired the most accurate results within most price movements. We recommend to use it,

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especially within the steep stable price course, and also when the price evolution changes significantly.

- **stable price increase**

Within the steep stable price course, the increase rate of the forecast stock exchanges was higher, in comparison with the increase rate of the stock exchanges during the approximation term. Therefore the stock exchanges increased faster than the calculated prognoses. Thus the prognoses at the end of the forecasting term contained the absolute percentage errors which caused the initial condition drift. When the price evolution in the forecasting term was stable, the longest initial condition drift was the most advantageous and the nearest to the stock exchanges.

In the forecasting term *October 2003 – March 2004* (Figure 4), the initial condition drift was caused by the prognosis in the third month of the term (December 2003, the percentage prognosis error was -8,24 %). In the next calculated prognoses after drift to the limiting month, the absolute percentage error was lower than 10 %, and all critical forecasting values were eliminated. By means of drift to the limiting month, the mean absolute percentage error of the forecasting term decreased from 9,44 % to 4,98 %.

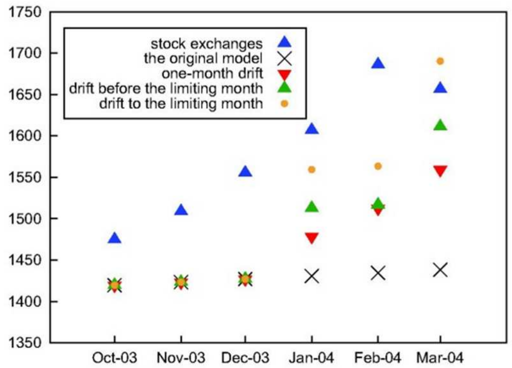

Figure 4 Forecasting success of drift to the limiting month within October 2003 – March 2004 (variant B)

- **steep price decline after price increase**

There was steep price decrease from April 2005 to June 2005. Within this period, the price decrease was significant compared to the previous increase. The stock exchanges in approximation terms were increasing, the approximation functions had also an increasing course, and the prognoses calculated by the original model were increasing too, so they were not sufficient to accommodate to a steep decline of the stock exchanges. The longer was the decline period, the higher was the absolute percentage error of the prognosis. Drift to the limiting month was the most successful within all forecasting terms with the price decline.

The forecasting term A*pril 2005 – September 2005* (Figure 5) was an example of the period when the original forecasting failed. Although the stock exchange decline occurred in April 2005, the highest decline was seen in May 2005. The prognosis absolute percentage error in this month exceeded the limiting value error 7 %, which caused the initial condition drift. At the longest drift, the initial condition value approached the prices in decline. Thus, unlike at shorter drifts, the critical value in June 2005 was eliminated. Since the stock exchanges in the next period increased, the forecasting by drift to the limiting month remained successful and no further corrections were needed. The absolute percentage error of the prognosis in May 2005 was at least 10 %, and within the forecasting term just one critical value occurred. After having used the longest initial condition drift, the remaining four critical values were eliminated, and a significant improvement of the forecasting was obtained. The mean absolute percentage error of the forecasting term decreased from 12,55 % to 4,96 % (variant B) and from 12,63 % to 4,94 % (variant E).

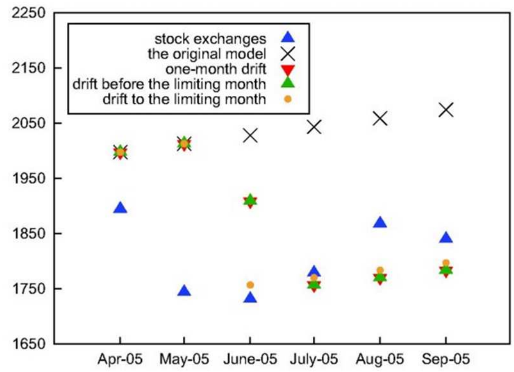

Figure 5 Forecasting success of drift to the limiting month within April 2005 – September 2005 (variant E)

- **price increase after price decline**

The end of the year 2005 and the first half of the year 2006 appeared as the most problematic. The problems in forecasting were caused by the steep increase of the stock exchanges after their important decline. The approximation terms with the price decline belonged to the mentioned forecasting terms. The higher was the number of the decline prices in approximation term, the slower was increase of the approximation function. Its course could be even decreasing. As these approximation functions served for the price prognoses within the rapid increase periods, the original forecasting being highly inaccurate. Slower increasing prognoses obtained lower values than quicker increasing stock exchanges; this is why the forecasting accuracy was decreasing with time (critical values applied to the months at the end of the forecasting terms). The forecasting was the most accurate when using the longest drift. By applying it, we put prognoses the nearest to the steep increasing stock exchanges.

Within the forecasting term *September 2005 – February 2006* (Figure 6), the initial condition drift occurred in the month with a steeper increase of the stock exchange (November 2005). Thanks to a moderate increase, the prognosis absolute percentage error in the limiting month was lower than 10 %. By using drift to the limiting month, the increase was captured and the prognoses for the following months were more accurate. Thus in the next steep increase period, by repeating the longest initial condition drift, the absolute percentage errors of the prognoses were lower than 10 %. MAPE decline in the forecasting term moved from 12,55 % to 6 %. Using the drift to the limiting month, all three critical values of the original model were eliminated.

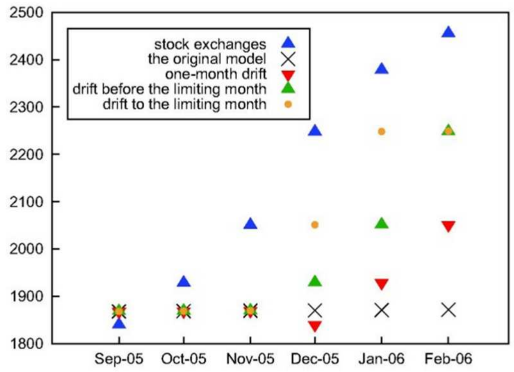

Figure 6 Forecasting success of drift to the limiting month within September 2005 – February 2006 (variant E)

3.3 The forecasting success of shorter initial condition drifts

If within a forecasting term, a more significant price fluctuation appeared, the most accurate was the drift that allowed placing the initial condition value the nearest to the real stock exchanges evolution. Thus, at the fluctuating stock exchanges, shorter drifts were more advantageous, namely one-month drift and drift before the limiting month. Using the longest drift the initial condition value was often replaced by the local maximal or minimal values that had caused the initial condition drift, which was not advantageous for forecasting following unstable price course.

The advantage of one-month drift was seen at the steep fluctuating increase with the significant price decline in the last month within the forecasting term *January 2006 – June 2006* (Figure 7). The first initial condition drift caused by the prognosis in the second month of the term (February 2006, the prognoses with the percentage error -7,35 % (variant B) or -7,49 % (variant E) were obtained). Therefore the absolute percentage error of the prognosis in the fourth month of the term was reduced to under 10 % (the original prognoses in April 2006 had the percentage error -12,18 % (variant B) and -12,43 % (variant E)). Using shorter drifts, the absolute percentage error of the prognosis in the fourth month was higher (at least 8 %), which caused the next initial condition drift, unlike at the longest initial condition drift. While the first initial condition drift was identical for both shorter drifts, the second drift allowed to obtain different initial condition values. The initial condition value for forecasting the price in the fifth month had the value of the stock exchange either in February 2006 (one-month drift) or in March 2006 (drift before the limiting month). But the stock exchange in March 2006 decreased, which was disadvantageous for forecasting steeply increased stock exchange in the fifth month, May 2006. This steep increase resulted in getting the critical value for all chosen drifts. Its following drift made the forecasting using one-month drift more accurate again. The price decline in the third month of the forecasting term (March 2006), which was the one-month drift initial condition value, was suitable for forecasting the price in the last month of the forecasting term, where the significant decline was observed. Conversely, the initial condition value using the drifts before the limiting month and to the limiting month was determined by significantly higher stock exchanges, which were not suitable for forecasting the price in decline. MAPE of the observed period decreased by the most advantageous one-month drift from 9,20 % to 6,17 % (variant B) and from 9,43 % to 6,26 % (variant E). The number of the critical values in both variants was reduced from two in the original model to one critical value.

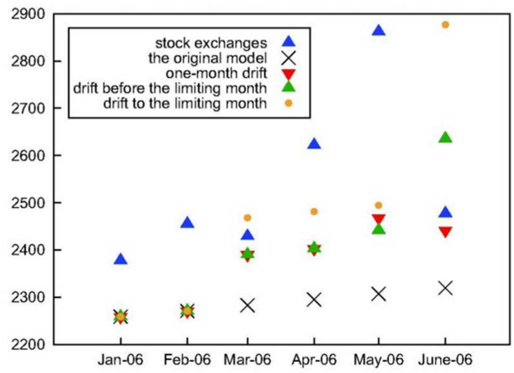

Figure 7 Forecasting success of one-month drift within January 2006 – June 2006 (variant E)

Drift before the limiting month was the most successful only in one forecasting term, *January 2004 – June 2004* (Figure 8). Within this term both observed variants B and E obtained the same results for all determined lengths of the initial condition drift because of the same approximation term in both variants. The period was characterized by moderate fluctuating increase. Within the period, the increase rate of the forecast stock exchanges was higher than the increase rate of the stock exchanges in the approximation term. Therefore the stock exchanges increased faster than calculated prognoses. The initial condition drift was caused by the prognosis in the month with local maximal aluminium price (the fourth month, April 2004, the percentage prognosis error was

-8,30 %). Since in the following months, the price increase did not continue, to the contrary the stock exchanges decreased, the initial condition drift to appropriate values was the most suitable. In this case it was the drift before the limiting month, because the stock exchange in the third month of the forecasting term, March 2004, was the closest to the stock exchanges at the end of the forecasting term. MAPE of the forecasting term was improved from 4,81 % to 4,23 %. As a result of a moderate increase, the critical value did not occur within the forecasting term.

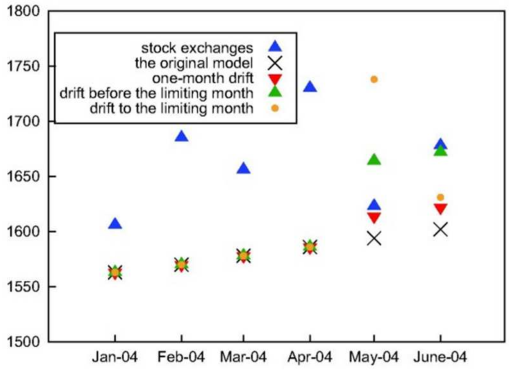

Figure 8 Forecasting success of drift before the limiting month within January 2004 – June 2004 (varianty B and E)

Conclusions

By having analyzed the forecasting success rate of the commodity price by means of initial condition drifts with different lengths, it was found out that the most accurate prognoses were the most often acquired when using the longest initial condition drift, drift to the limiting month. We recommend to use this initial condition drift type, especially at stable increase, or decrease and during larger changes in price evolution.

Within moderate price increase or decrease, the original forecasting was mostly so accurate (the absolute percentage prognosis errors were lower than 7 %), that the initial condition drift did not occur. When within the observed period the price fluctuations appeared, the longest drift was not always the most accurate. The drift replacing the initial condition value by the stock exchange the nearest to the next price course was the most accurate. For this reason, within significant price fluctuations we recommend to use shorter initial condition drifts where the initial condition value did not acquire local maximal or minimal values.

By the most appropriate types of the initial condition drift the original forecasting was significantly improved and the strategy of initial condition drift contributes to prognoses accuracy.

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